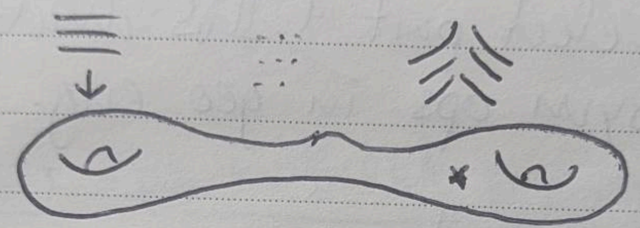
w/ L. Bhardwaj, S. Schäfer-Nameki,
S. Giacomelli

Punctures and Symmetries in class S

$C_{g,n}$, $\mathfrak{g} = \mathfrak{su}(N)$, Higgs φ

$\bar{\partial}_A \varphi = \partial_A \bar{\varphi} = 0 \qquad F + [\varphi, \bar{\varphi}] = 0$

$\Sigma \subset T^* C_{g,n} : \det(\lambda - \varphi) = 0$



N-sheeted covering

" N M5-branes wrapped on $C_{g,n}$ "

4d $\mathcal{N}=2$ SQFT, CB: Casimirs of φ

Q: Defect & sym ops from Σ ?

Interesting because:

1. Non-Lagrangian characterization
2. Deformations to $\mathcal{N}=1$ & $\mathcal{N}=0$

Timely because:

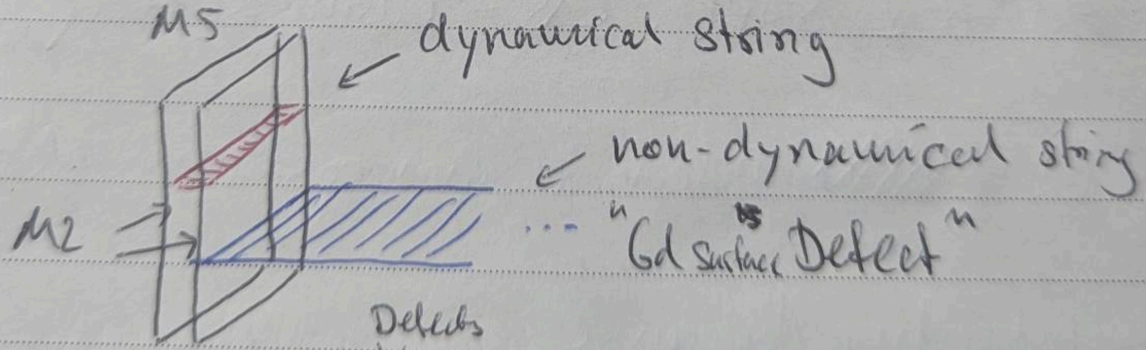
1. Defect part partially understood
2. Recent string / M-th constructions for symmetry ops in geo engineering
3. Class S various geo duals IIB

Outline: Defect part + IIB dual, Sym ops. in geo eng.

Defect Ops in flat space:

M-theory:

$\mathbb{R}^{1,5} \times \mathbb{R}^5$, N M5s on $\mathbb{R}^{1,5} \times \{0\}$



Goal: track Defects through compactification

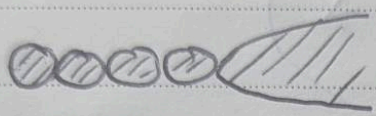
IIB: $\mathbb{R}^{1,5} \times X_N$, $X_N = \mathbb{C}^2/\mathbb{Z}_N$

M2 $\to$ D3

D3 wrapped on vanishing curves
→ dynamical strings

D3 " non-compact curves → Defects

$H_2(X_N, \partial X_N) \cong \mathbb{Z}_N$

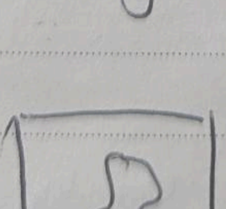


<u>Defect Ops without punctures</u>

No punctures: M5s on C_g

$\Rightarrow$ 4d line defects: $\mathcal{L} = H_1(C_g) \otimes \mathbb{Z}_N = \mathbb{Z}_N^{2g}$

4d Lagrangian gauge theory ~~cases~~: W, H



Ex. $C_g = T^2$: 4d $N=4$ SYM,

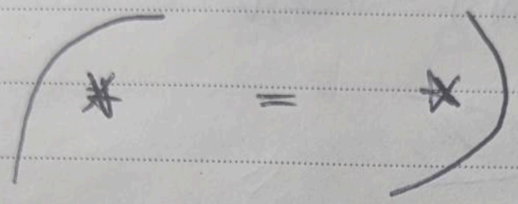
$\mathcal{L} = \mathbb{Z}^{(A)}_{G=SU(N)} \oplus \mathbb{Z}^{(B)}_{G=SU(N)}$

other ADE ...

<u>Defect Ops</u> <u>with</u> regular <u>punctures</u>

$$\varphi \sim N \frac{dz}{z} + \text{regular}$$

$C_{g,n}$:



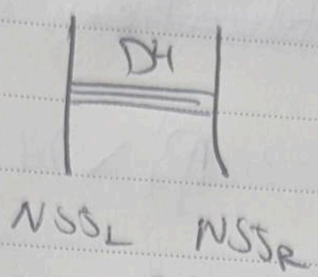
~~Lines do not~~
~~Regular punch~~

Lagrangian Examples : Reg punches are transparent
~~A on A~~

<u>Defect Ops. w/</u> <u>irregular punctures</u>

Motivation : IIA

	0	1	2	3	4	5 6 7 8 9
$ND4$	✓	✓	✓	✓	$[X^4_L, X^4_R]$	~~✗~~
$NS5_L$	✓	✓	✓	✓	X^4_L	✗ ✗
$NS5_R$	✓	✓	✓	✓	X^4_R	✗ ✗



4d $\mathcal{N}=2$ SYM

M-theory lift : x^{10} circle

$t = \exp(-x^4 - i x^{10})$ $\quad v = x^5 + i x^6$

Brane system $\to$ single M5 on $\mathbb{R}^{1,3} \times \Sigma'$

Σ' : $\frac{1}{t} + t = P_N(v) = v^N + \varphi_2 v^{N-2} + \dots + \varphi_N$

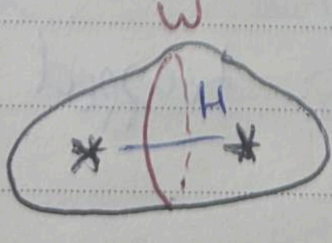
$\mathbb{P}^1_t$, punctures $t = 0, \infty$ $\quad \sim \det(\lambda - \varphi)$

$\lambda = v \frac{dt}{t}$

$\varphi \sim \frac{1}{t^{1/N}} \operatorname{diag}(1, \omega, \omega^2, \dots, \omega^{N-1}) \frac{dt}{t}$

+ regular

Field theory : $L = \mathbb{Z}_N \oplus \mathbb{Z}_N$



Defects can end on punctures

No.
Date / /

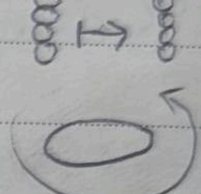
IIB dual : ALE ~~fibration~~ fibered CY3

$$\widetilde{\mathbb{C}^2/\mathbb{Z}_N} \hookrightarrow Y_6 \to \mathbb{P}^1_t \setminus \{t=0,\infty\}$$

Boundary model

$$\widetilde{\mathbb{C}^2/\mathbb{Z}_N} \hookrightarrow Y_5 \to S^1$$

Monodromy :



$$M_p : \Lambda_{\text{roots}} \to \Lambda_{\text{roots}}$$

$$\left(\Lambda_{\text{weights}} \to \Lambda_{\text{weights}}\right)$$

D3-branes on non-cpct 2-cycless

$$\text{Tor}\, H_2(Y_5) \cong \text{coker}(M_p - 1) \cong \mathbb{Z}_N$$

Regular puncture $M_p = 1$

Generic punctures : trapped lines

Interlude: Isolated Hypersurface Sing.

IHS type puncture $P_{b,k}$ on $C_{0,1}$

$\xrightarrow[\text{CB tuning}]{\text{Scaling limits}}$ Isolated hypersurface sing $X_{b,k}$

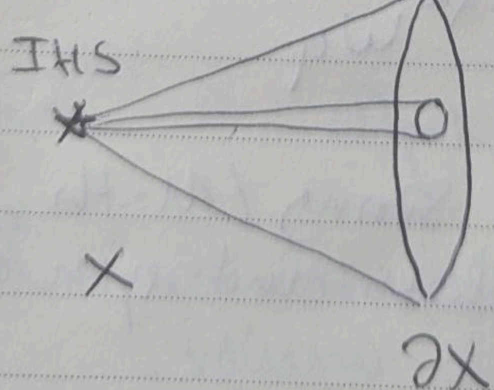
$\mathfrak{g}$ = ADE, Argyres-Douglas type

$$\varphi = \frac{1}{z^{1+k/b}} \cdots$$

Ex: $(A_{k-2N+1}, D_N) \subseteq D = \mathfrak{g}$

$X_{2N-2,k}$:
$$x_1^2 + x_2^{N-1} + x_2 x_3^2 + x_4^{k-2N+2} = 0$$

$2N-2$



$\gamma \in \mathrm{Tor}\, H_2(\partial X)$

$\updownarrow$

$\mathrm{Tor\ coker}\,(M_{P_{2N-2,k}} - 1)$