Symmetries, Geometry & Gravity

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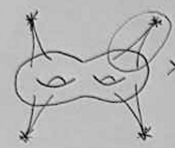
① Summary & Outlook : M/IIA/IIB on $\mathbb{R}^{\#} \times X \longrightarrow S_X$ (non-compact)

$X = K3, T^n/\Gamma, \text{TCS } G_2, \ldots$



Sing: X_i^{loc} } QFT $\mathcal{T}_{X_i^{loc}}$ w/ Generalized Global Symmetries

$\mathcal{T}_{X_1^{loc}} \otimes \ldots \otimes \mathcal{T}_{X_N^{loc}} \rightarrow S_X$

Global Symmetries $\rightarrow \varnothing$

Example : M-th on $T^4/\mathbb{Z}_2$, A_1^{16}

$A_1 \sim \mathbb{C}^2/\mathbb{Z}_2$, 7D $SU(2)^{16}$ SYM

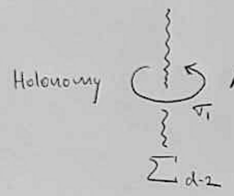
② Field Theory

G gauge theory on $\mathbb{R}^{1,d-1}$ w/ gravity

Center $Z_G \subset G$

Topological Gukov-Witten Ops.:

$GW_z(\Sigma_{d-2}) \qquad z \in Z_G$



Gaiotto-Kapustin-Seiberg-Willett '14

Top. Operators $\leftrightarrow$ Symmetry

$GW_z(\Sigma_{d-2}) \circlearrowleft W_{R_G}(\gamma_1)$

(Wilson lines)

$\langle GW_z(\Sigma_{d-2}) W_{R_G}(\gamma_1) \ldots \rangle =$

$e^{i\#} \langle W_{R_G}(\gamma_1) \ldots \rangle$ 1-form Symmetry

W/ Gravity : Gravitational Solitons

$\longrightarrow$ New electric States

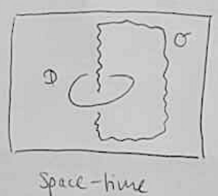
Rep (G/Z_G)

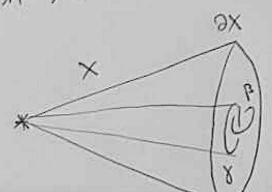
$\Rightarrow$ 1-form symmetry unbroken

Generalized Global Symmetries in String Theory

M/IIA/IIB on $\mathbb{R}^{\#} \times X \xrightarrow{\text{non-compact}} \mathcal{T}_X$

$X = \mathbb{C}^n/\Gamma$, NHCs, $\mathcal{S}(S^3/\Gamma), \ldots$





Symmetry Operators : Branes wrapped on asymptotic cycles

$$\mathcal{O}_k \equiv H_k(\partial X)$$

Defect Operators : Branes wrapped on non-compact relative cycles

$$\mathbb{D}_k \equiv \frac{H_k(X, \partial X)}{H_k(X)} \cong H_{k-1}(\partial X)\Big|_{triv}$$

Example : M-th on $X = \mathbb{C}^2/\mathbb{Z}_N$

$\partial X = S^3/\mathbb{Z}_N$, $\mathcal{T}_X$ = 7D $SU(N)$ SYM

$$H_k(S^3/\mathbb{Z}_N) = \begin{cases} \mathbb{Z} & k=0 \\ \mathbb{Z}_N = \langle \gamma \rangle & 1 \\ 0 & 2 \\ \mathbb{Z} & 3 \end{cases}$$

M2 on cone(γ) $\sim W_F$

M5 on ~~cone~~ γ $\sim GW_{z_\gamma}$

Remarks:

- Non-Lagrangian characterization of defect & sym ops.
- Streamlined prescription & Generalizations
- Computationally useful

Example : D-branes

$$\sigma(\mathcal{M}) = \int [DA_i]_\sim \exp\left(2\pi i \int_{M \times \beta} \mathcal{L}_{top}^{Dp}\right)$$

World volume TFT
$\rightarrow$ Non-inv fusion

$$\int_{M \times \beta = \mathcal{M}} \exp(F_2 - B_2) \sqrt{\frac{\hat{A}(T\mathcal{M})}{\hat{A}(N\mathcal{M})}} \oplus C^{RR}_{\text{odd/even}}$$

Applications : QFT

- Generalized Landau Paradigm
- Symmetry TFT, anomalies

$SU(N) \rightarrow$ | TFT | ($SU(N)$, Top. B.C.)

- ABJ anomalies, BSM

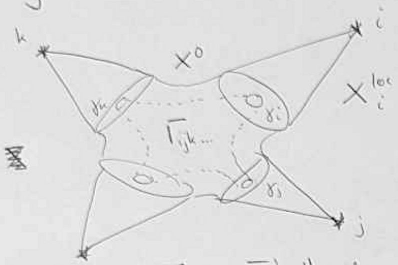
String Theory

- High-dim SCFTs, Junctions
- $\mathcal{N}=0$ String constructions
- Swampland

⑤ GGS & No Global Symmetry

X compact

$$X = X^{loc} \cup X^0$$
$$X^{loc} \cong \sqcup_i X_i^{loc}, \qquad X^0 \cong X \setminus X^{loc}$$



$$\partial \Gamma_I = \sum_{i \in I} \#_i \gamma_i$$

Sym. Ops.: $L_k : H_k(\partial X^{loc}) \to H_k(X^0)$

Defect Ops.: $\partial_k : H_k(X) \to H_{k-1}(\partial X^{loc})$

$\rightsquigarrow$ Mayer-Vietoris sequence:

$$\ldots \to H_k(X) \xrightarrow{\partial} \underbrace{H_{k-1}(\partial X^{loc}) \to H_{k-1}(X^{loc}) \oplus H_{k-1}(X^0)}_{} \to \ldots$$

Exactness $\equiv$ No Global Symmetries

Case Study: M-th on $X = K3$

ADE singularities @ pts, $\mathbb{C}^2/\Gamma_i$, S^3/Γ_i

$\longrightarrow \mathcal{T}_{X^{loc}} = 7D$ $\mathfrak{g} = \oplus_i \mathfrak{g}_i$ SYM

1-form & 4-form symmetries

MV subsequence:

$$0 \to H_2(X^0) \to H_2(X) \to H_1(\partial X^{loc}) \to H_1(X^0) \to 0$$

$$H_1(X^0) \overset{PL}{\cong} H^3(X^0, \partial(X^0)) \overset{Ex}{\cong} H^3(X, \{x_i\}) \cong H^3(X)$$

$$\xrightarrow{UCT} \mathrm{Tor}\, H_1(X^0) = (\mathrm{Tor}\, H_2(X))^\vee$$

$T^4/\mathbb{Z}_2$:

$$0 \to \mathbb{Z}^6 \to \underset{\text{Identified}}{\mathbb{Z}^6} \oplus \underset{\text{Gauged}}{\mathbb{Z}_2^5} \to \mathbb{Z}_2^{16} \to \underset{\text{Broken}}{\mathbb{Z}_2^5} \to 0$$

$$G = \frac{\frac{SU(2)^{16}}{\mathbb{Z}_2^5} \times U(1)^6}{\mathbb{Z}_2^6}$$

Non-Geometric Formulation