ON THE HOLOGRAPHIC DUAL OF A TOPOLOGICAL SYMMETRY OPERATOR

SYMMETRY SEMINAR
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BASED ON 2401.09538 w/ J.J. HECKMAN, C. MURDIA

AdS/CFT: Any operator of the boundary CFT should have a bulk counterpart

Generalized global symmetries: Symmetries of QFTs $\Longleftrightarrow$ topological operators

$\Longrightarrow$ Do bulk counterparts of topological symmetry operators exist? What are their properties?

String Holography: Topological operators "come to life" in the bulk as the topological sector of dynamical branes

[ABBS '22], [GE '22], [BLW '23], [ABGS '23], ...

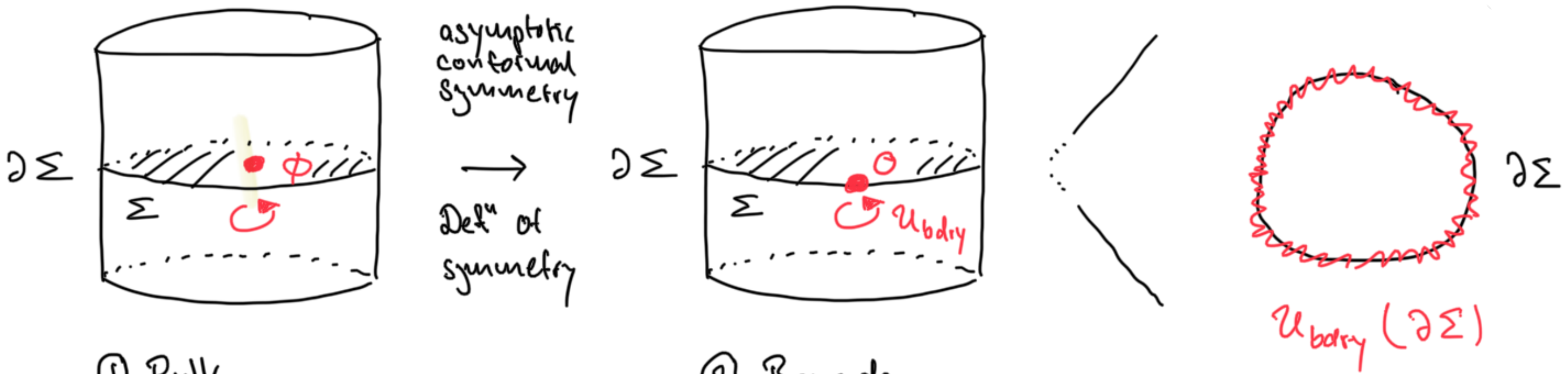
GOAL: Bottom up explanation, without reference to top down construction

CONSEQUENCE: No-global-symmetries conjecture

Review: Harlow & Ooguri 2018

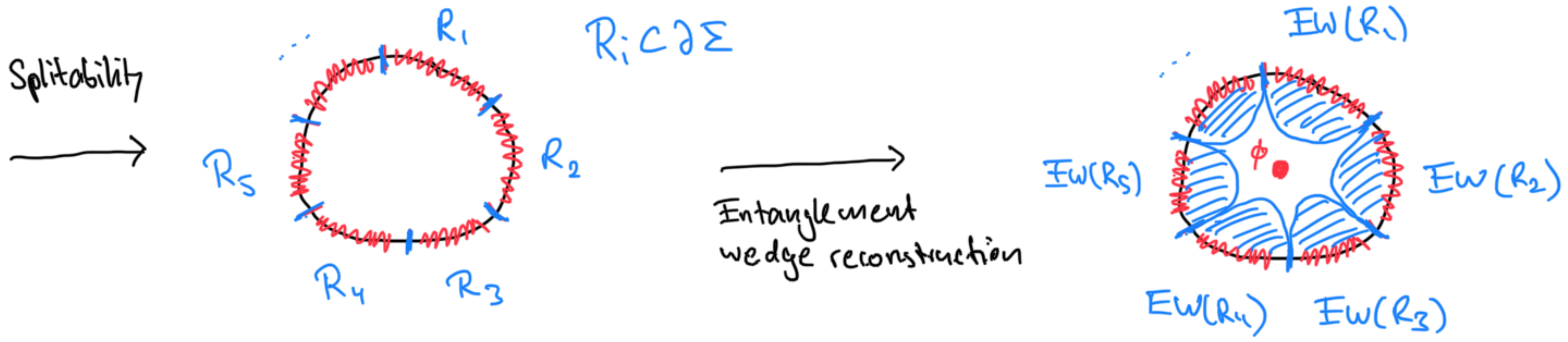
"No quantum gravity theory in asymptotically AdS space which has a global symmetry can be dual to a boundary conformal field theory."

Restrict to $CFT_{D\geq 2}$ & invertible 0-form sym.



① Bulk

② Boundary



$$U_{bdry}(\partial\Sigma) = \prod_i U_{bdry}(R_i)$$

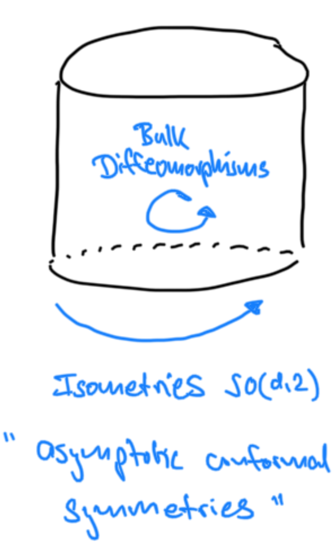
③ Sub region

④ Contradiction

$$\text{supp}\,\phi \not\subset \bigcup_i EW(R_i)$$

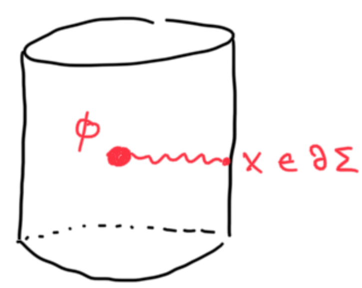
Restrict discussion internal 0-form sym.

① Bulk



Physical local operators are :

- invariant under bulk diffeomorphisms ⇝ Gravitational Dressing
- in representations of $SO(d,2)$



Internal Global symmetries :

- act faithfully on {gravitationally dressed ϕ}
- preserve the bdry pt x
- ...

bulk tensor index
→ boundary tensor index

Definition is such that :

global symmetry of a holographic asymptotically AdS quantum gravity theory ⇘ global sym. of the dual conformal field theory

Comments:
- Invertible p-form symmetries (For CFT_D $D \geq p+2$)
- Non-invertible symmetries

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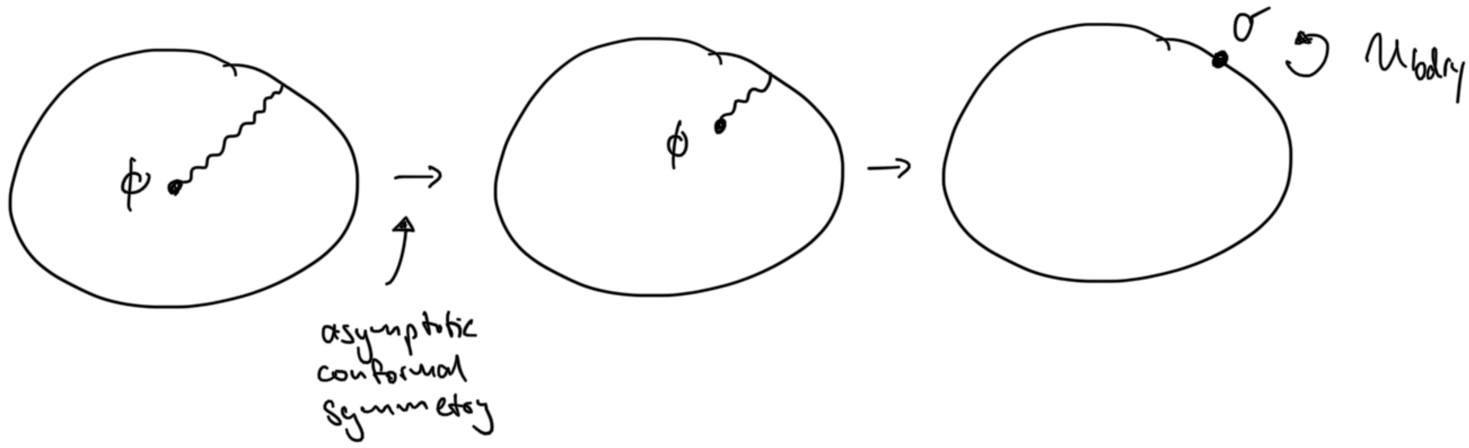
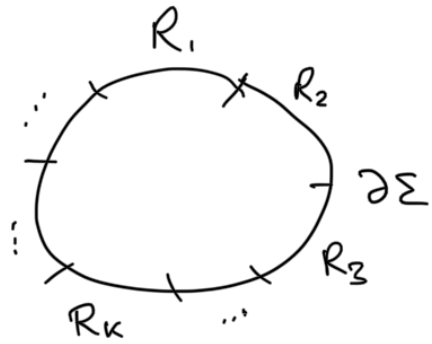
(1) Bulk → (2) Boundary



comments:
- no discussion of bulk objects generating symmetries

Splittability: (2) Boundary → (3) Subregion ⊆ Boundary



Motivation: Noether Currents

$$\mathcal{U}(\underbrace{e^{i\varepsilon^a T_a}}_{g}, \partial\Sigma) = \exp\left(i\varepsilon^a \int_{\partial\Sigma} *j_a\right)$$

$\partial\Sigma = \bigcup_K R_K$

$$= \prod_K \exp\left(i\varepsilon^a \int_{R_K} *j_a\right) = \prod_K \mathcal{U}(e^{i\varepsilon^a T_a}, R_i)$$

Def^n: A global symmetry of a QFT on $\mathbb{R}_t \times M$ is splittable on M if for every open spatial subregion $R \subseteq M$ there exist unitary operators $V(g,R)$ s.t.

$$V^\dagger(g,R)\, \mathcal{O}\, V(g,R) = \begin{cases} \mathcal{U}^\dagger(g,M)\, \mathcal{O}\, \mathcal{U}(g,M) & \forall\, \mathcal{O} \in A(R) \\ \mathcal{O} & \forall\, \mathcal{O} \in A(\mathrm{Int}[M-R]) \end{cases}$$

Comments:
- spacetime dependent
- ABJ Anomaly $\rightsquigarrow$ $\mathbb{Z}_{N_f}$ not splittable on some M
- $V(g,R)$ are not topological

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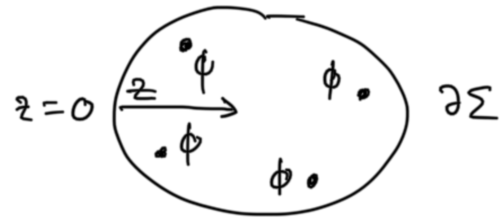
(4) [illegible] (in AdS)

(4) Entanglement wedge reconstruction (in AdS)

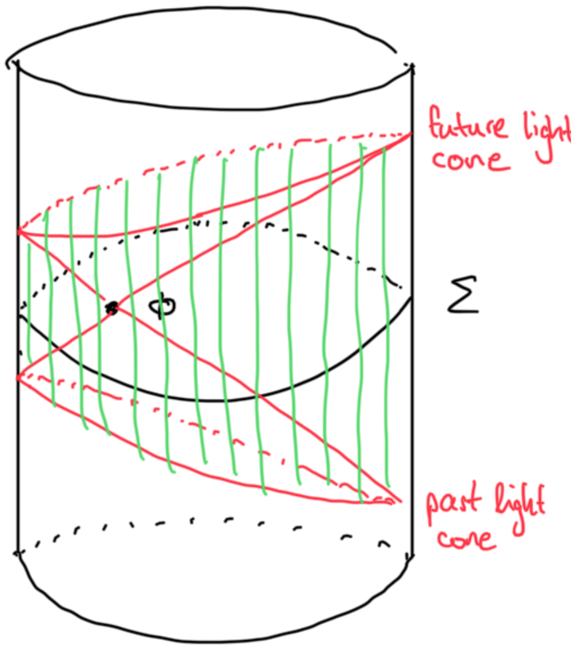
$\boxed{4a}$ Extrapolate Dictionary [BDHM '98]



$$\lim_{z\to 0} z^{-n\Delta} \langle \phi(x_1,z) \dots \phi(x_n,z) \rangle_{bulk}$$

$$= \langle \mathcal{O}(X_1) \dots \mathcal{O}(X_n) \rangle_{CFT}$$

$\boxed{4b}$ HKLL Reconstruction ['06]



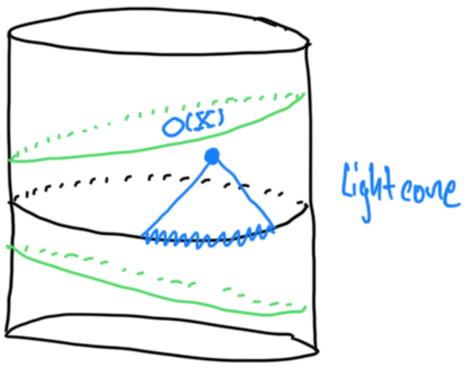
(solve eqs radially)

$$\phi(x) = \int_{S_x} dX \; K(x;X)\, \mathcal{O}(X)$$

Green's functions / Smearing kernel

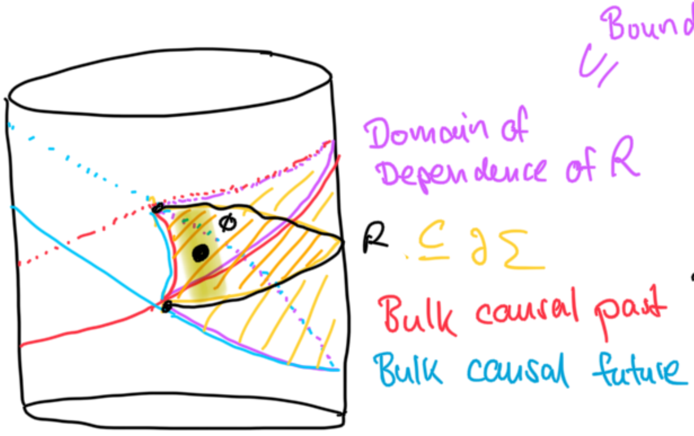
Improve S_x to a domain contained in a const. time slice:

$$\mathcal{O}(X) = \mathcal{O}(x,t) \xrightarrow[\text{evolve}]{\text{time}} F(\mathcal{O}(x,0))$$



⟶ Bulk Operators admit reconstruction from ONE time slice

$\boxed{4c}$ Rindler Reconstruction [HKLL '06]



subregion duality

↓

operators admit reconstruction from R if contained in Entanglement wedge

EW(R)

=

Intersection of

Motivation: Green's function $K(X,x)$ not unique

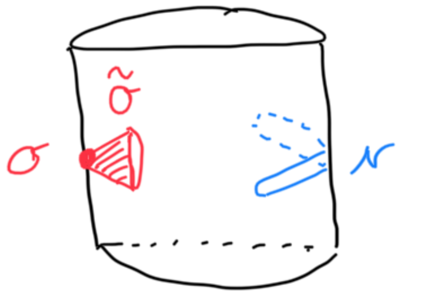
$\exists\, \mathcal{L}(X;x)$ s.t. $K(X,x) + \mathcal{L}(X,x)$ also Green's function

use to kill support of K in $\partial\Sigma - R$

Consequence:

$\mathcal{U}(g, R)$ implements global symmetry on all operators in $EW(R)$ and does not act on operators in $EW(R^c)$.

BULK DUAL OF A TOPOLOGICAL SYM OPERATOR? (No-global-syms in asymp. AdS?)



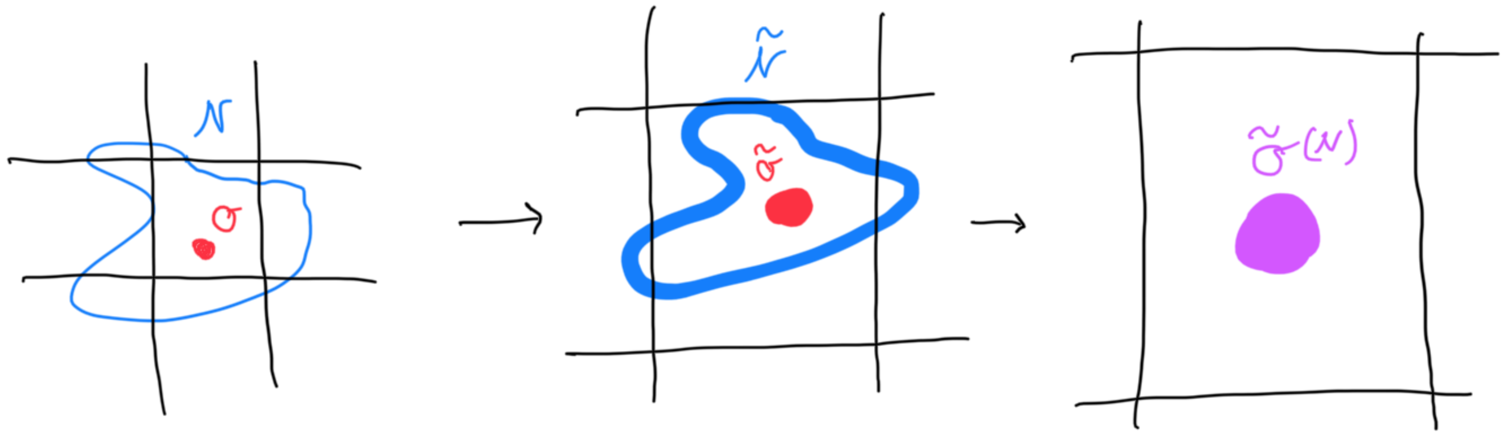
CFT: $\mathcal{N}_Y \sim \int [da] \exp\left(2\pi i \int_Y \mathcal{L}_{TFT}\right)$

Sym. Action : $\mathcal{N}$: $\mathcal{O} \mapsto \mathcal{O}^{(\mathcal{N})}$

Bulk : Bulk reconstruct of $\mathcal{O}$: $\tilde{\mathcal{O}}(x,z) \sim \int dX\, \mathcal{O}(X)\, K(X; x, z)$

BULK DUAL OF $\mathcal{N}$? DOES $\tilde{\mathcal{N}}$ EXIST? IS $\tilde{\mathcal{N}}$ TOPOLOGICAL?

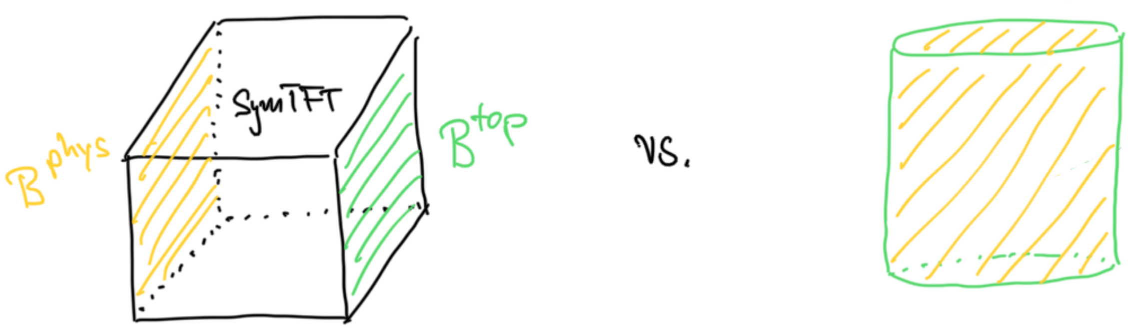
Holographic RG flow:



$z = 0$ | small $z > 0$ | $z > z_*$

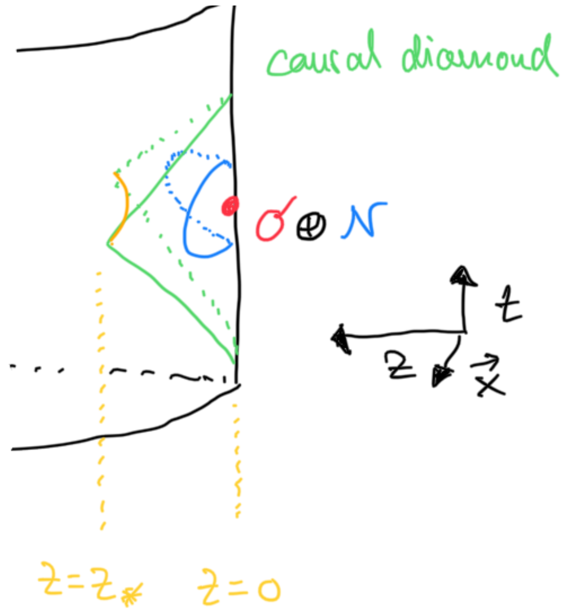
Comments: • Sym. action: $\tilde{\mathcal{O}}$ and putative $\tilde{\mathcal{N}}$ link in each radial slice

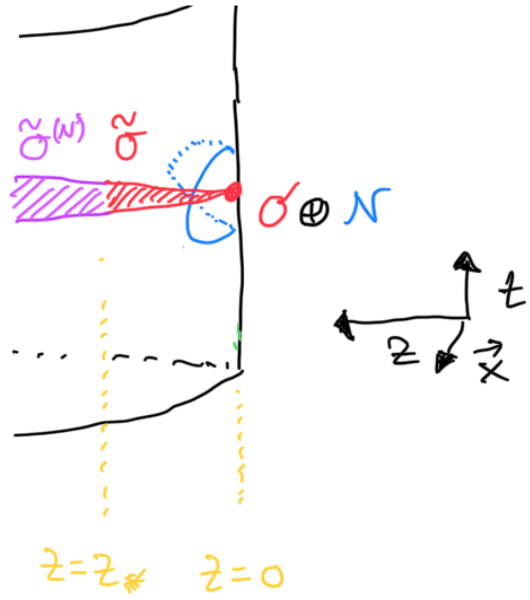
$\leftarrow$



• Entanglement Wedge Reconstruction / Subregion duality

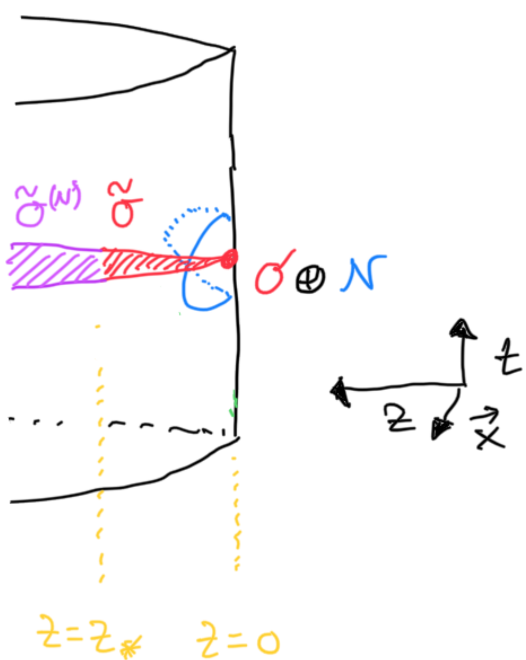
$\Rightarrow z_*$ finite

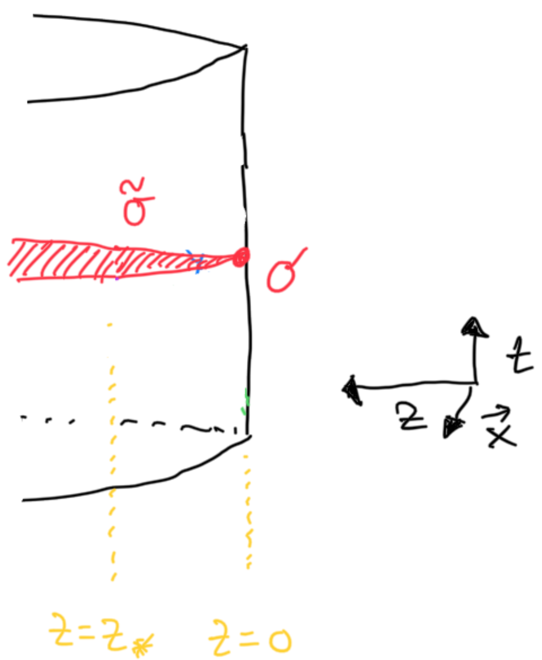




CLAIM: $\tilde{N}$ exists & is not topological

Compare the configurations





(1) $\tilde{\sigma} \mapsto \tilde{\sigma}^{(N)}$

Stress-energy localized near $z=z_*$

(2) $\tilde{\sigma} \mapsto \tilde{\sigma}$

No additional stressenergy

$\leadsto$ $\tilde{N}$ needs to exist to account for difference in stress-energy & $\tilde{N}$ is not topological

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Constraint: $\tilde{N} \xrightarrow{z \to 0} N$

$$\Rightarrow \tilde{N}(z) \sim Z_1 \times \int [da] \exp\left(2\pi i \int \mathcal{L}_{TFT} + \text{Non-topological} \right)$$

J

$Y(z)$

$\uparrow$ homotopic to $Y(0)$ $= \text{supp}\, \mathcal{N}$

<u>Example</u>: topological sector of $\tilde{\mathcal{N}}$ from SymTFT

$U(1)^{(0)}$: $\mathcal{N}_\eta^{CFT_D} = \exp\left(2\pi i \eta \int_{Y_{D-1}} j_{D-1}\right)$ w/ background a_1

$\tilde{\mathcal{N}}_\eta^{top}(z) = \exp\left(2\pi i \eta \int_{Y_{D-1}(z)} F^{dual}\right)$ w/ $F^{dual} = * dA_1$

$dF^{dual} = J_D \Rightarrow \tilde{\mathcal{N}}(z=0) = \exp\left(2\pi i \eta \int_{C_D} J_D\right) = \exp\left(2\pi i \eta \int_{Y_{D-1}^{(0)}} j_{D-1}\right)$

$\partial C_D = Y_{D-1}(0)$

<u>Estimates</u>:

$$S_{brane} \sim \tau_q \int_Y d^q\xi \sqrt{\det M} + \text{topological terms}$$

D-branes: $M_{ij} = h_{ij} + b_{ij} + 2\pi\alpha' F_{ij} + \ldots$

on dimensional grounds: $\tau_q \sim \frac{1}{\ell_*}$ Compton Wavelength of $\tilde{\mathcal{N}}$

$$\ell_{AdS} \gg \ell_* \gtrsim \ell_{pl}$$

argument was in grav EFT

rate of smearing

<u>SUMMARY</u>:
- No global symmetries in asymptotic AdS
- Symmetry operator in CFT_D $\Rightarrow$ "Brane" in gravitational bulk

<u>OMISSIONS</u>:
- (-1)-form & Lower form symmetries
- Splittability revisited