ON THE HOLOGRAPHIC DUAL OF A TOPOLOGICAL SYMMETRY OPERATOR

SWAMPLAND SEMINAR
FEBRUARY 27th

MAX HÜBNER UPENN

BASED ON 2401.09538 w/ J.J. HECKMAN, C. MURDIA

Holographic Dictionary : Any operator of the boundary CFT should have a bulk counterpart

Generalized global symmetries : Symmetries of QFTs $\Longleftrightarrow$ topological operators

$\Longrightarrow$ Do bulk counterparts of topological symmetry operators exist ? What are their properties ?

String Holography : Topological operators "come to life" in the bulk as the topological sector of dynamical branes

[ABBS '22], [GE '22], [BLW '23], [ABGS '23], ...

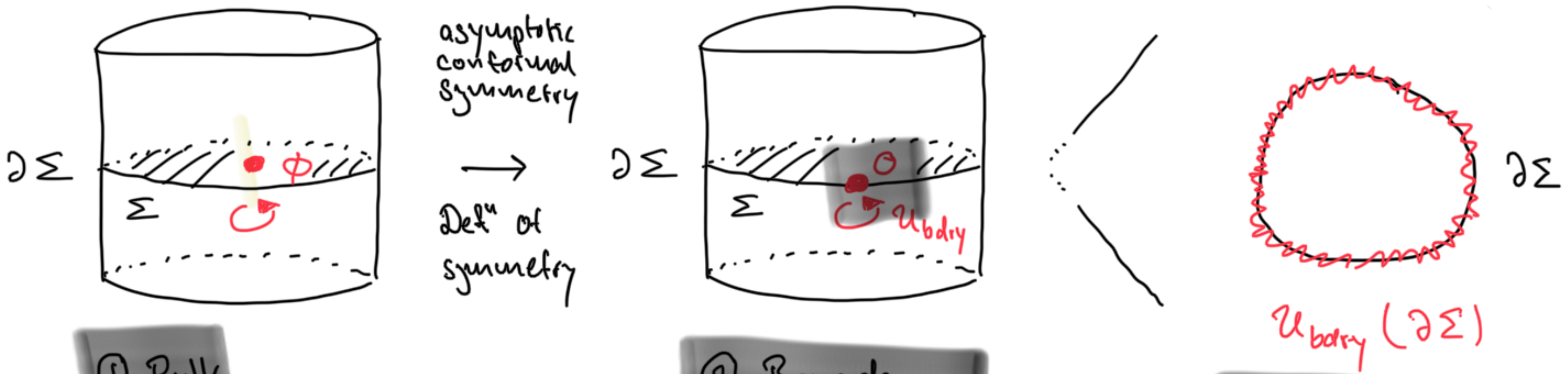
GOAL : Bottom up explanation, without reference to top down construction

CONSEQUENCE : No-global-symmetries

Review: Harlow & Ooguri 2018

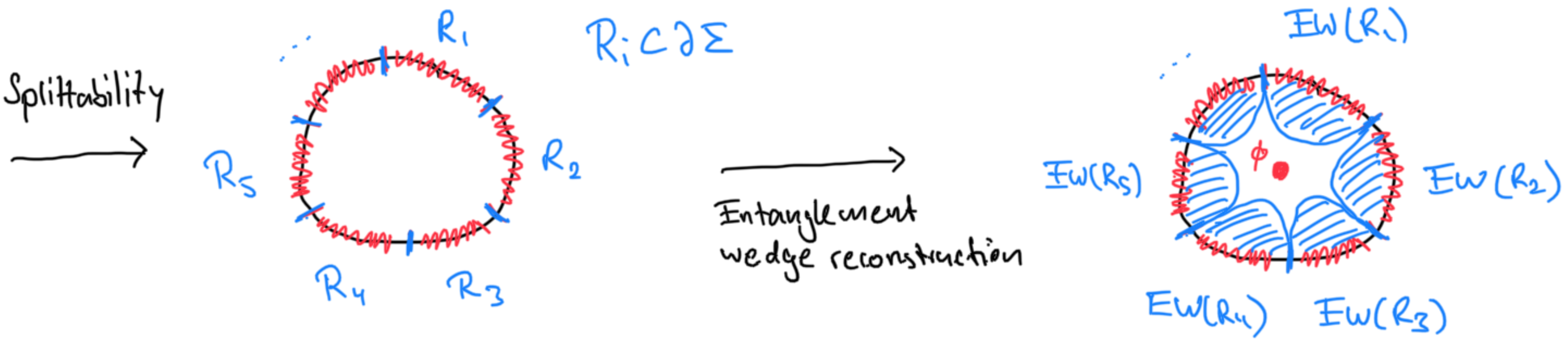
"No quantum gravity theory in asymptotically AdS space which has a global symmetry can be dual to a boundary conformal field theory."

Restrict to $CFT_{D\geq 2}$ & invertible 0-form sym.



① Bulk

② Boundary



$\mathcal{U}_{bdry}(\partial\Sigma) = \prod_i \mathcal{U}_{bdry}(R_i)$

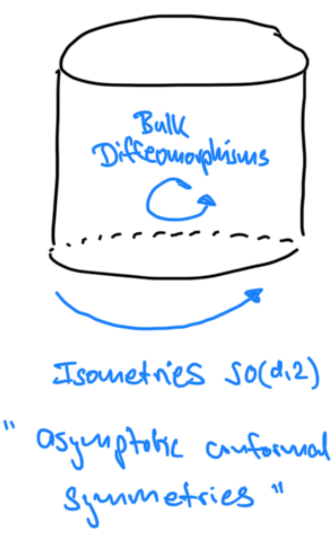
③ Subregion $\subseteq$ Boundary

④ Subregion $\subseteq$ Bulk

Contradiction: $\mathrm{supp}\,\phi \not\subset \bigcup_i EW(R_i)$

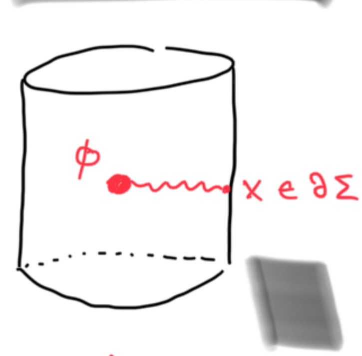
Restrict discussion internal 0-form sym.

① Bulk ⟶ ② Boundary



Physical local operators are :

- invariant under bulk diffeomorphisms ⟶ Gravitational Dressing
- in representations of SO(d,2)

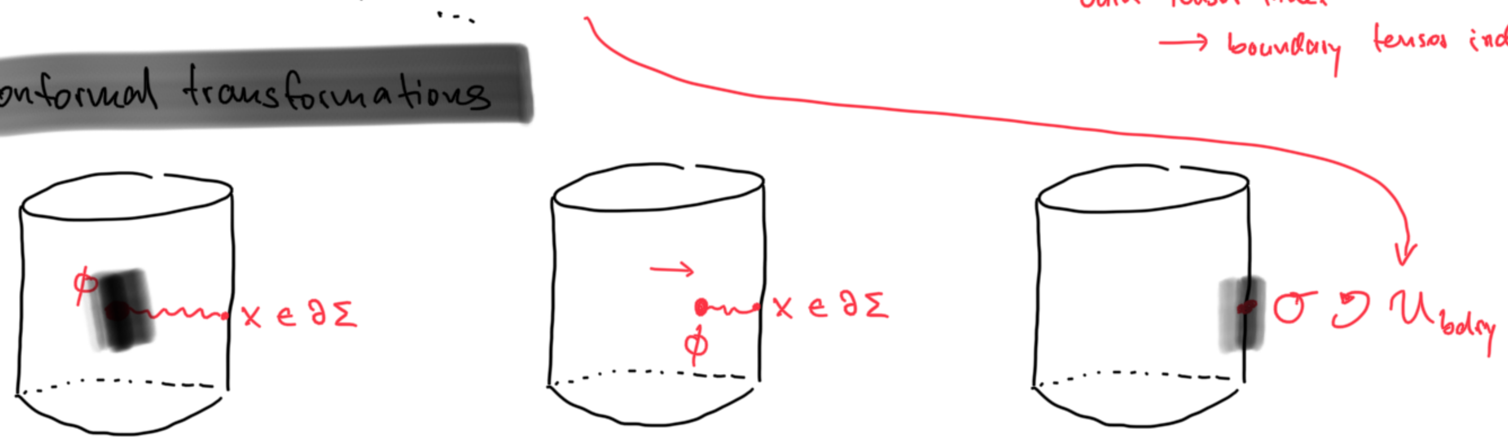


Internal Global symmetries :

- act faithfully on {gravitationally dressed ϕ}
- preserve the bdry pt x
- ...

bulk tensor index ⟶ boundary tensor index

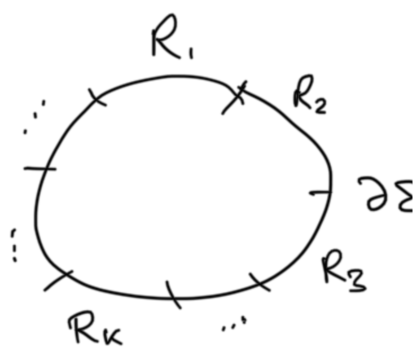
Conformal transformations



Fully retreat onto Bdry: Global symmetry of a holographically asympt. AdS quantum gravity theory $\Longrightarrow$ Global symmetry of CFT

Comments:
- Invertible p-form symmetries (For CFT_D $D \geq p+2$)
- Non-invertible symmetries
- no discussion of bulk objects generating symmetries

Splittability: ② Boundary $\longrightarrow$ ③ Subregion $\subseteq$ Boundary



Motivation: Noether Currents

$$\mathcal{U}(\underbrace{e^{i\varepsilon^a T_a}}_{g}, \partial\Sigma) = \exp\left(i\varepsilon^a \int_{\partial\Sigma} * j_a\right)$$

$\partial\Sigma = \bigcup_K R_K$

$$= \prod_K \exp\left(i\varepsilon^a \int_{R_K} * j_a\right) = \prod_K \mathcal{U}(e^{i\varepsilon^a T_a}, R_i)$$

Def^n: A global symmetry of a QFT on $\mathbb{R}_t \times M$ is splittable on M if for every open spatial subregion $R \subseteq M$ there exist unitary operators $V(g, R)$ s.t.

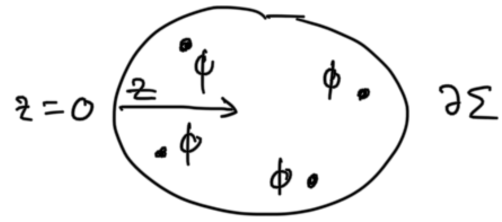
$$V^\dagger(g,R)\, \mathcal{O}\, V(g,R) = \begin{cases} \mathcal{U}^\dagger(g,M)\, \mathcal{O}\, \mathcal{U}(g,M) & \forall\, \mathcal{O} \in A(R) \\ \mathcal{O} & \forall\, \mathcal{O} \in A(\mathrm{Int}[M-R]) \end{cases}$$

Comments:
- non-inv. symmetries generically not splittable
- spacetime dependent
- ABJ Anomaly $\rightsquigarrow$ $\mathbb{Z}_{N_f}$ not splittable on some M
- $V(g,R)$ are not topological

---

(4) Entanglement wedge reconstruction (in AdS)

$\boxed{4a}$ Extrapolate Dictionary [BDHM '98]



$$\lim_{z\to 0} z^{-n\Delta}\langle \phi(x_1,z)\dots\phi(x_n,z)\rangle_{bulk}$$

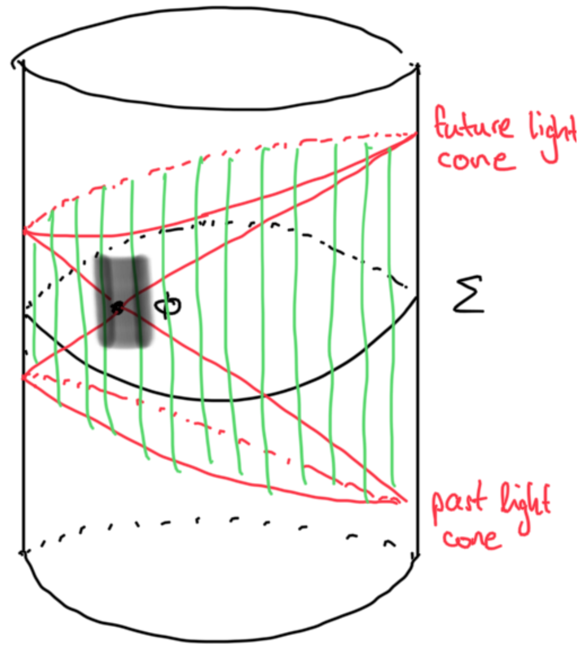
$$=\langle \mathcal{O}(X_1)\dots\mathcal{O}(X_n)\rangle_{CFT}$$

$\boxed{4b}$ HKLL Reconstruction [`06]

("solve eqs radially")

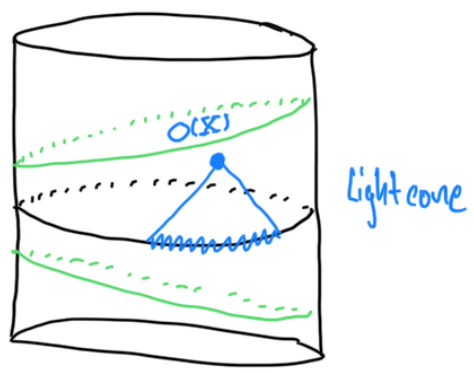
$$\phi(x) = \int_{S_x} dX \; K(x;X)\,\mathcal{O}(X)$$

Green's functions / Smearing kernel



Improve S_x to a domain contained in a const. time slice:

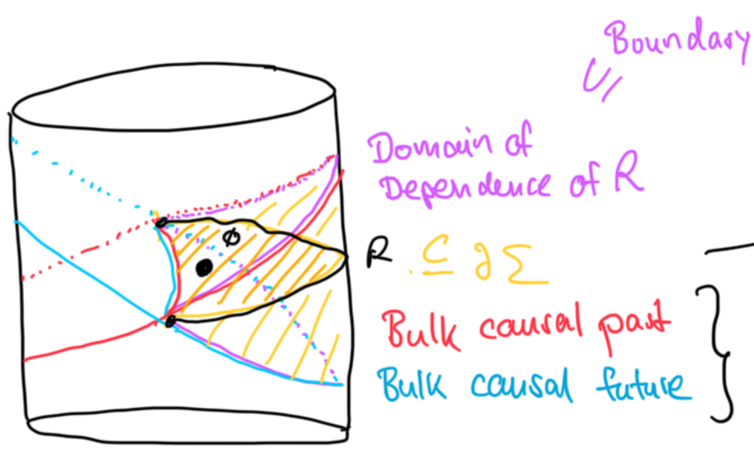
$$\mathcal{O}(X) = \mathcal{O}(x,t) \xrightarrow[\text{evolve}]{\text{time}} F(\mathcal{O}(x,0))$$



$\longrightarrow$ Bulk Operators admit reconstruction from ONE time slice

$\boxed{4c}$ Rindler Reconstruction [HKLL '06]

Subregion duality



operators admit reconstruction from R if contained in Entanglement wedge

EW(R)
=
Intersection of

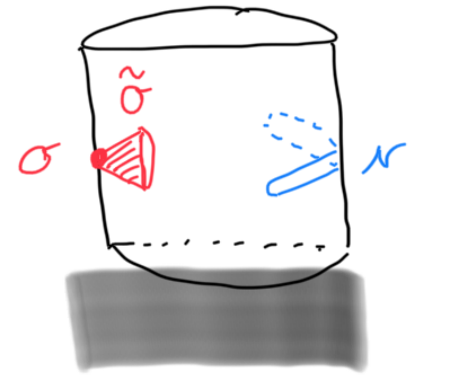
Motivation: Green's function $K(X,x)$ not unique: $K \longrightarrow K'$

use to kill support of K in $\partial\Sigma - R$

Consequence:

$\mathcal{U}(g,R)$ implements global symmetry on all operators in $EW(R)$ and does not act on operators in $EW(R^c)$.

BULK DUAL OF A TOPOLOGICAL SYM OPERATOR? (No-global-syms in asymp. AdS?)



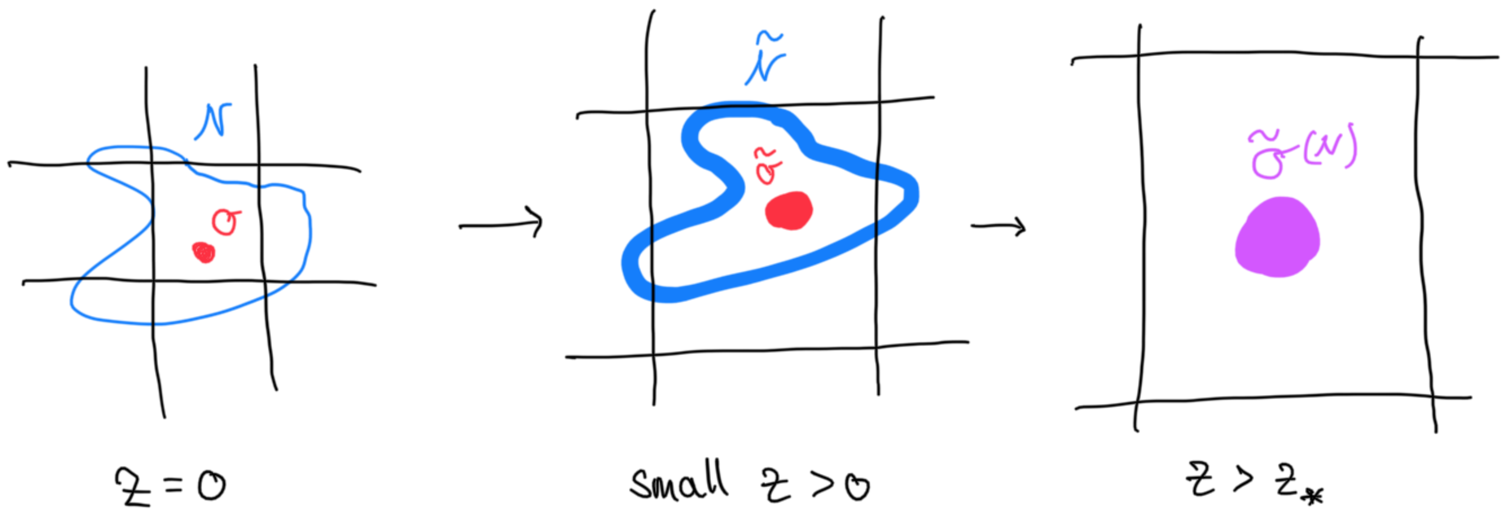
CFT: $\mathcal{N}_Y \sim \int [da] \exp\left(2\pi i \int_Y \mathcal{L}_{TFT}\right)$

Sym. Action: $\mathcal{N}$: $\mathcal{O} \longmapsto \mathcal{O}^{(\mathcal{N})}$

Bulk: Bulk reconstruct of $\mathcal{O}$: $\tilde{\mathcal{O}}(x,z) \sim \int dX\, \mathcal{O}(X)\, K(X; x,z)$

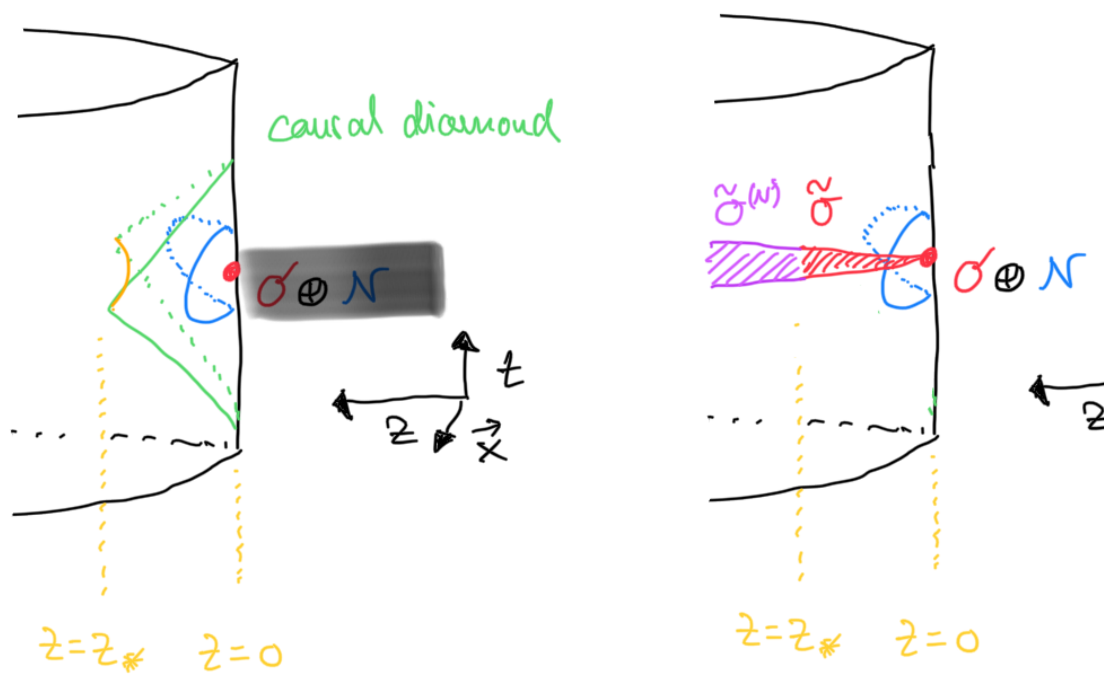
BULK DUAL OF $\mathcal{N}$? DOES $\tilde{\mathcal{N}}$ EXIST? IS $\tilde{\mathcal{N}}$ TOPOLOGICAL?

Holographic RG flow:



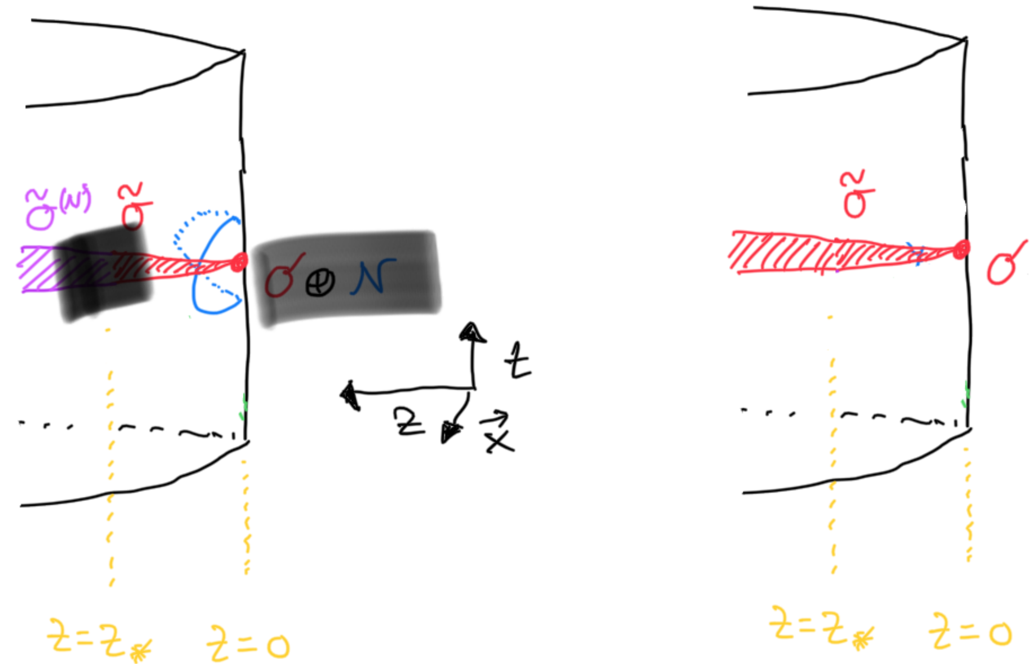
Entanglement Wedge Reconstruction / Subregion duality

$\Rightarrow$ z_* finite



CLAIM : $\tilde{\mathcal{N}}$ exists & is not topological (N positive codim.)

Compare the configurations



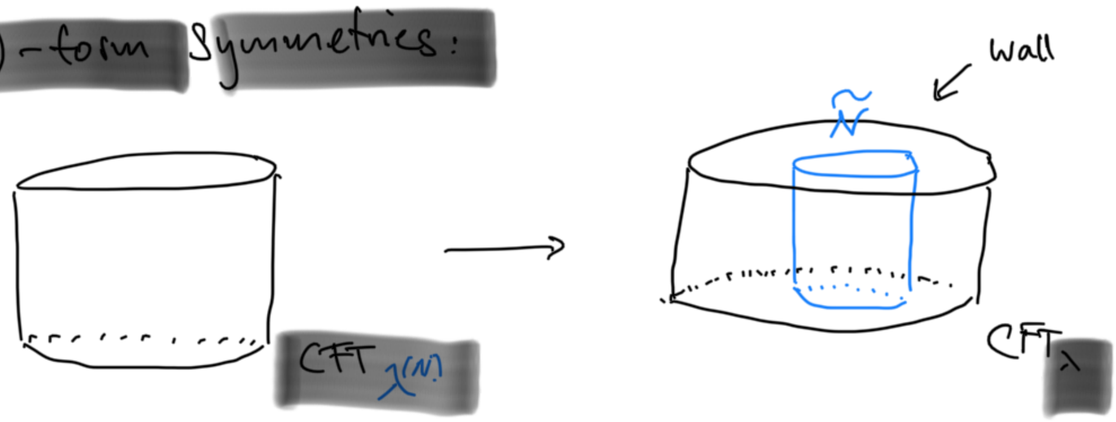
(1) $\tilde{\sigma} \mapsto \tilde{\sigma}^{(\mathcal{N})}$ (2) $\tilde{\sigma} \mapsto \tilde{\sigma}$

Stress-energy localized near $z = z_*$

No additional stress-energy

$\rightsquigarrow$ $\tilde{\mathcal{N}}$ needs to exist to account for difference in stress-energy & $\tilde{\mathcal{N}}$ is not topological

(-1)-form Symmetries:



bulk field ϕ w/ $\phi|_\partial \sim \lambda$

Constraint: $\tilde{\mathcal{N}} \xrightarrow{z \to 0} \mathcal{N}$

$$\Rightarrow \tilde{\mathcal{N}}(z) \sim Z_1 \times \int [da] \exp\left(2\pi i \int \mathcal{L}_{TFT} + \text{Non-topological}\right)$$

$Y(z)$

↑ homotopic to $Y(0)$ $=$ supp $\mathcal{N}$

Estimates:

$$S_{brane} \sim T_q \int_Y d^q\xi \sqrt{\det M} + \text{topological terms}$$

D-branes: $M_{ij} = h_{ij} + b_{ij} + 2\pi\alpha' F_{ij} + \dots$

on dimensional grounds: $T_q \sim \dfrac{1}{\ell_*}$

Compton Wavelength of $\tilde{\mathcal{N}}$

$$\ell_{AdS} \gg \ell_* \gtrsim \ell_{pl}$$

rate of smearing

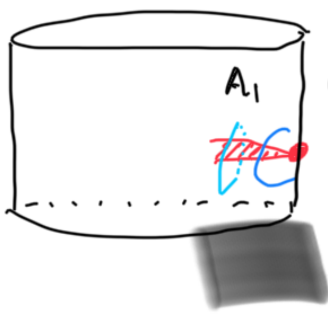
argument was in grav EFT

Example: Gukov-Witten operators

$U(1)^{(0)}$: $\mathcal{N}_\eta^{(bdry)} = \exp\left(2\pi i \eta \int_{Y_{D-1}} j_{D-1}\right)$

a_1 background

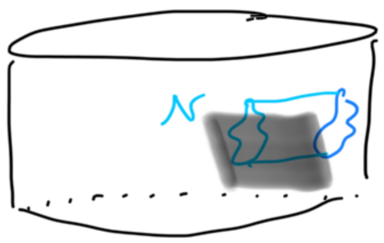
A_1 bulk $\longrightarrow F^{dual} = * dA$



$$\mathcal{N}_\eta^{top}(z) = \exp\left(2\pi i \eta \int_{Y_{D-1}(z)} F^{dual}\right)$$

$$\mathcal{N}_\eta^{(top)}(z) \xrightarrow{z=0} \mathcal{N}_\eta^{(bdry)} \quad : \quad dF^{dual} = J_D$$

Non-Abelian$^{(0)}$: $\mathcal{N}_{[g]}^{(bdry)} = \exp\left(2\pi i\, t^{cd} \int_{Y_{D-1}} j_{cd}\right)$ $\qquad d_A F^{dual} = J_D$



Genuine

Gukov-Witten operators are labelled by conjugacy classes!

SUMMARY:

- No global symmetries in asymptotic AdS (codim p top operators $p \geq 1$)
- Symmetry operator in CFT_D $\Rightarrow$ "Brane" in gravitational bulk

OMISSIONS:

- Lower form symmetries

- Splittability revisited