SYMTREES & SYMMETRIES

QUIVER MEETING
FEB 23rd 2023

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based on 2310.12980 & 2307.13027 in collaboration with
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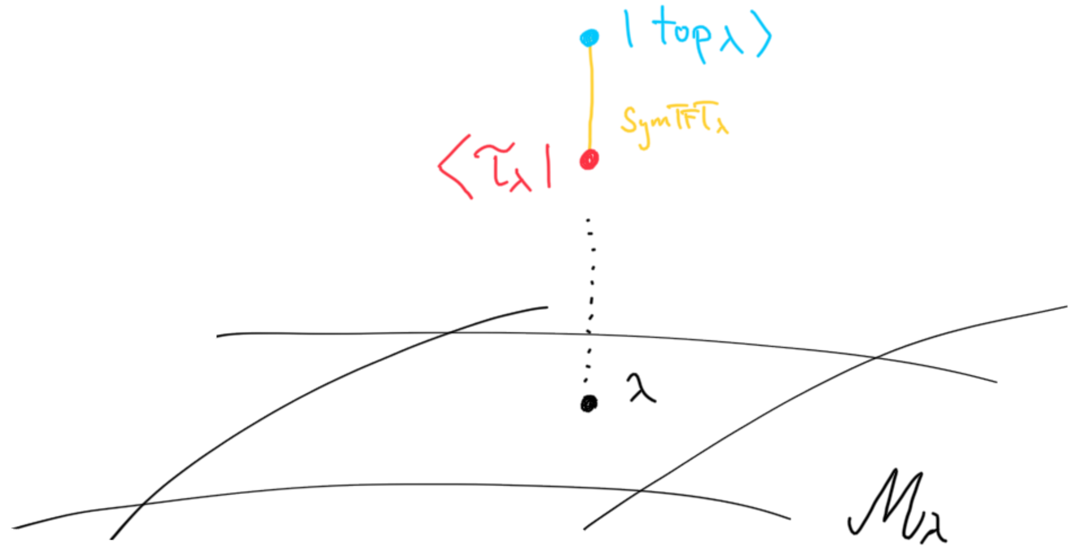
The SymTFT sandwich: Given a QFT $\mathcal{T}_\lambda$ consider [FMT22]

λ : Parameters $\rightsquigarrow$ Family of Sandwiches



Symmetries are topological: $\lambda \longrightarrow \lambda' = \lambda + \varepsilon$

$\mathrm{SymTFT}_\lambda = \mathrm{SymTFT}_{\lambda'}$

GOAL: Study Decoupling Limits with topological non-decoupling

REMARK : Other applications, no-global-symmetries in QG

[BBPS, 2023] Club Sandwich

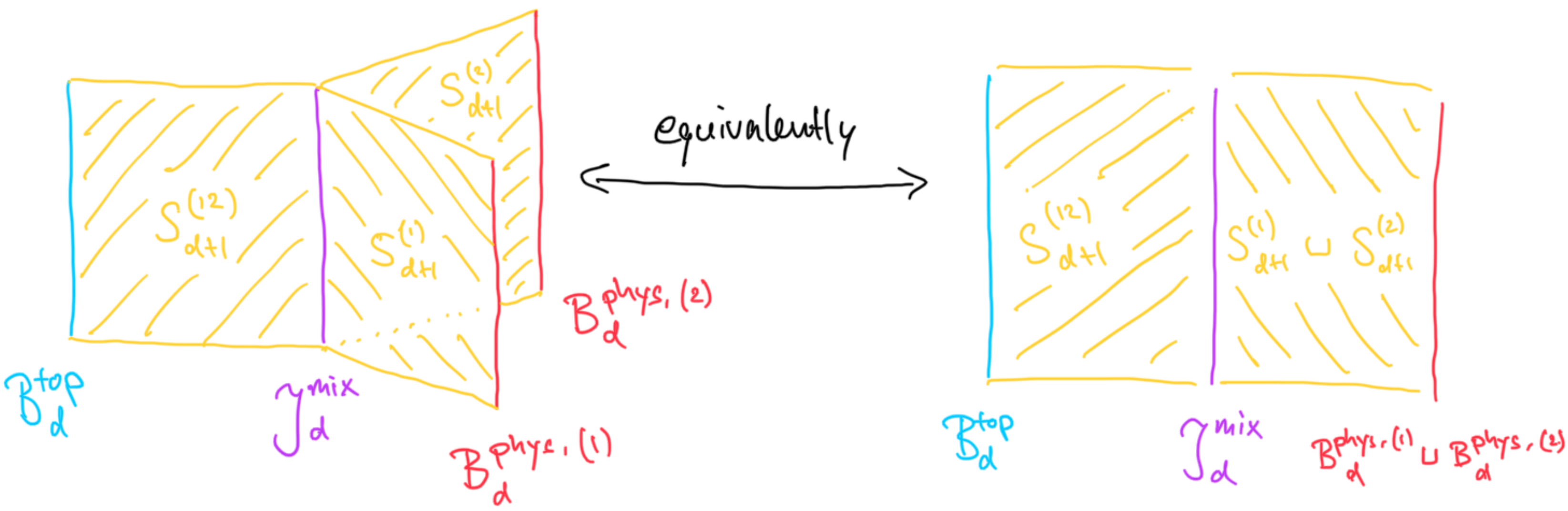
$\rightsquigarrow$ determine gapped infra-red phases of 2d theories with symmetry $\mathcal{S}$

WHAT IS A SYMTREE ?

- Collection of SymTFTs $\{ \mathcal{S}_{d+1} \}$
- Collection of Boundary Conditions $\{ \underbrace{\mathcal{B}^{\text{top}}_d , \mathcal{B}^{\text{phys}}_d}_{\text{BC for exactly one SymTFT}} , \mathcal{J}^{\text{mix}}_d \}$

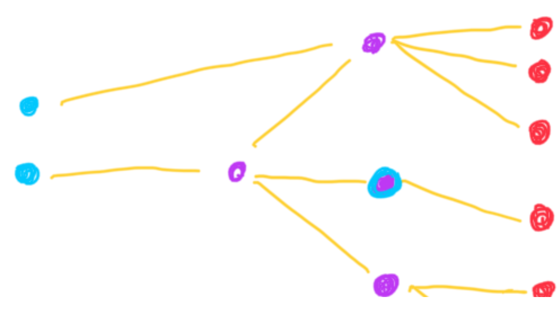
BC for multiple SymTFTs simultaneously $\downarrow$ $\mathcal{J}^{\text{mix}}_d$

Schematic Example :



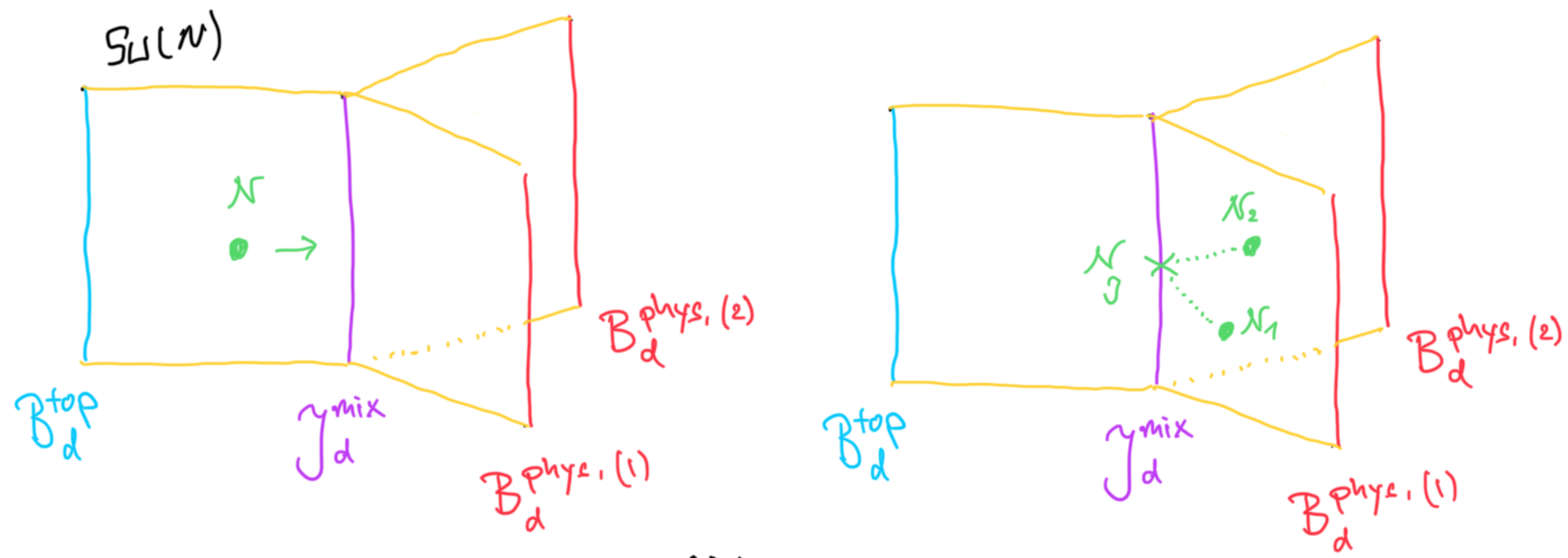
Comment :
- # top BC & # phys BC not fixed
- Junctions could be purely topological / physical
- Valency of Junctions is ≥ 2
- Today : Depth of Tree finite, no loops

Smaller sketch for convenience :

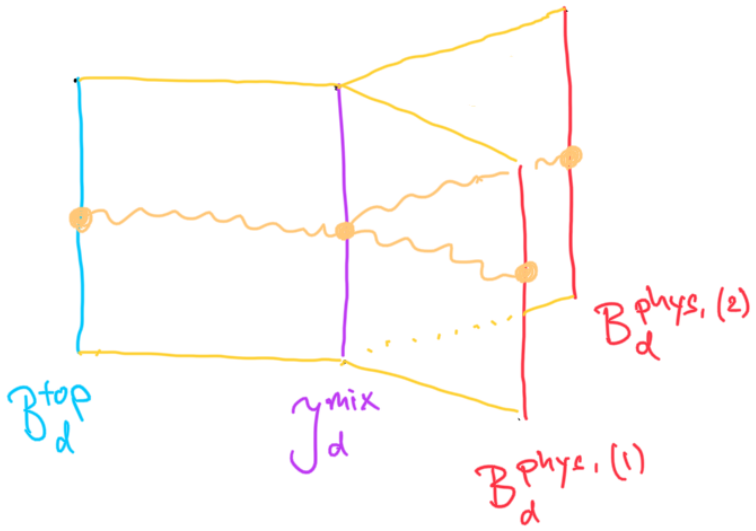
SymTree $\Longleftrightarrow$ 

DEFECT & SYMMETRY OPERATORS

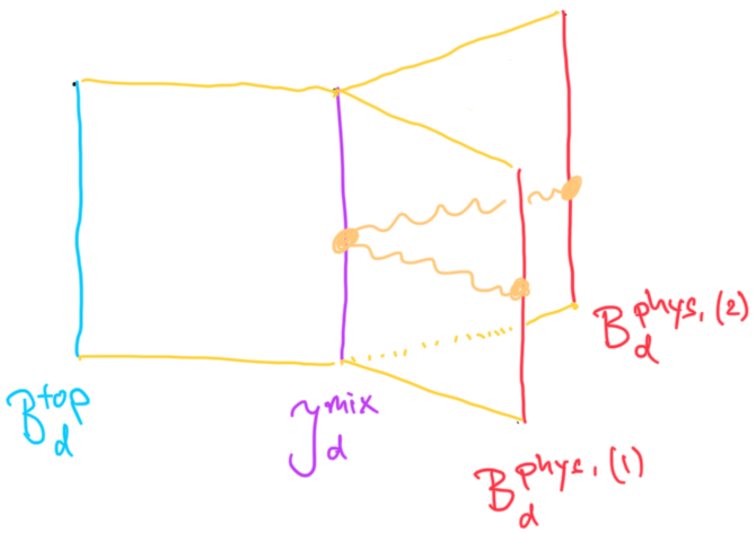
(Here for QFT SymTrees, i.e., one top boundary condition)



(i)



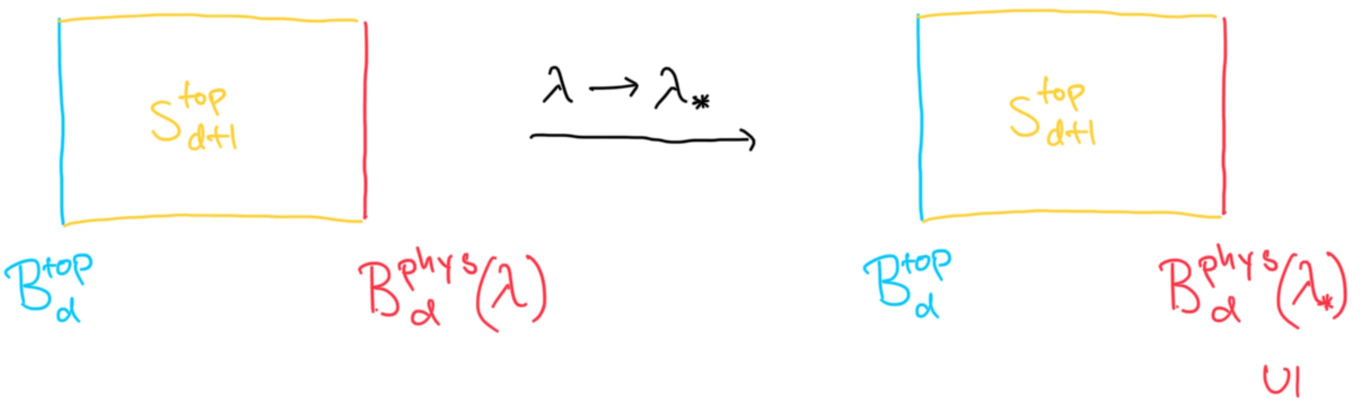
(ii)



(iii)

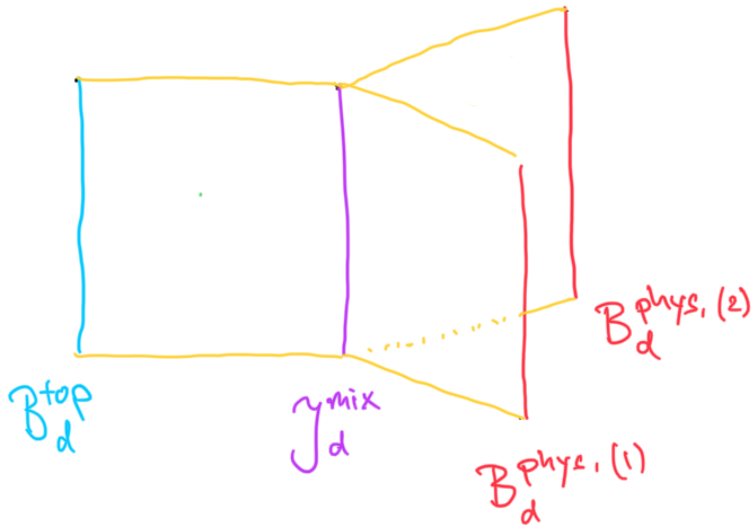
SYMTREE OPERATIONS

① "Growing roots"



$B_d^{phys,(1)}(\lambda_*) \sqcup B_d^{phys,(2)}(\lambda_*) \sqcup \ldots$

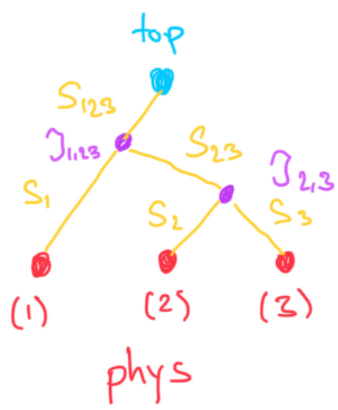
$\updownarrow \cong$



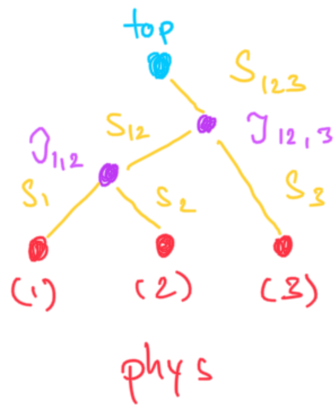
Example: theory with action S_Λ and suppression scale Λ s.t.

$$\Lambda \to \infty : \quad S_\Lambda \longrightarrow S_1 + S_2 + S_3 + S_{top}$$

② Associator / Anomalies



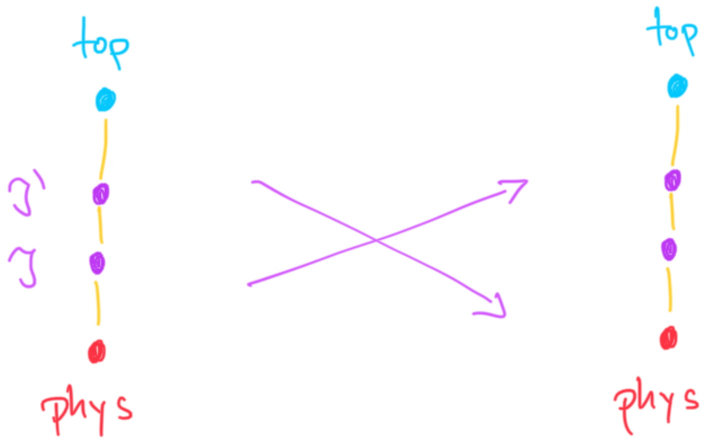
vs.



partition function

$$Z \to e^{i\alpha} Z$$

(eg. decoupling limits do not commute)

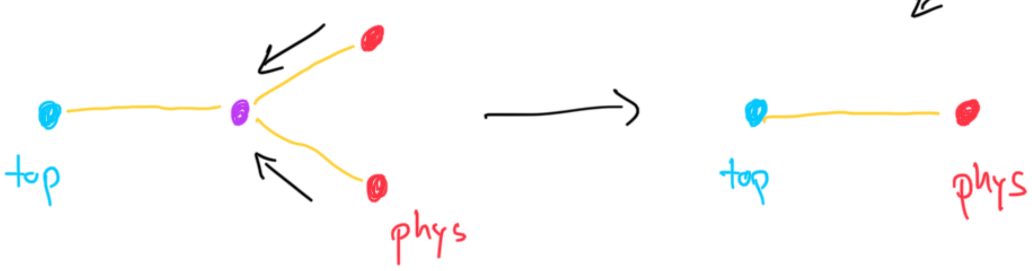


(treat SymTrees as topological operators in a trivial theory)

③ Standard Edge contraction

Sandwich Contraction:

Sandwich

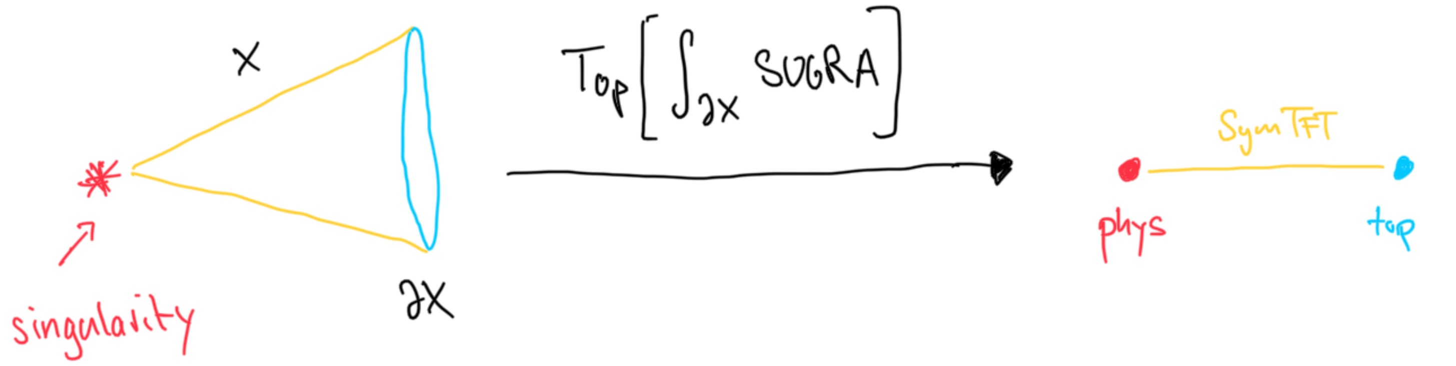


& others

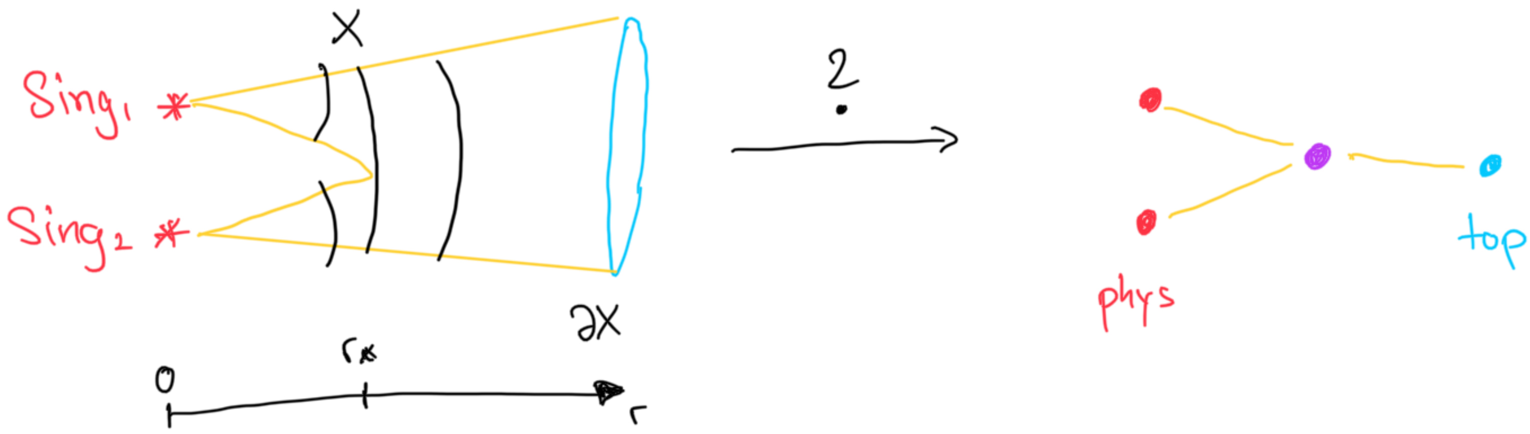
TOP DOWN APPROACH

well-known & powerful : [A B GE H S 2022]

IIA/IIB/M-th w/ purely geometric background $X = \mathrm{Cone}(\partial X)$

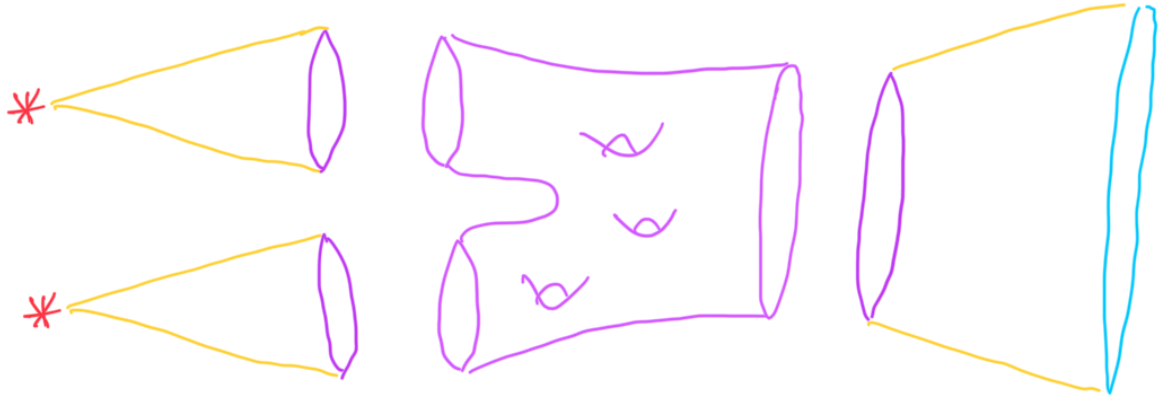


What about asymptotically conical X ?



Does $\mathrm{Top}\left[\int_{\partial X} \mathrm{SUGRA}\right]$ work ? Essentially "YES", however recall that the Junction can be partially physical.

Idea: "Pair of Pants" Decomposition

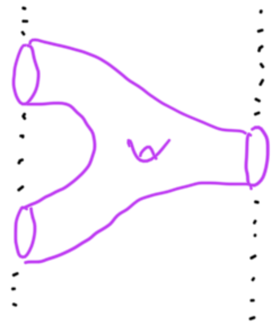


Formalized via Filtration $\mathcal{F}_X$ of Geometry X

↳ topologically piecewise constant

THE Junction

$V =$



Junction Fields : $H^*(V;\mathbb{Z})$

Some Junction fields mediate gluing conditions

$$H^*(\mathcal{U}_{r<r_*}) \longrightarrow H^*(V;\mathbb{Z}) \longleftarrow H^*(\mathcal{U}_{r>r_*})$$

Degrees of freedom fluctuating on Junction originate from

$$H^*(V;\mathbb{Z}) \big/ H^*(\mathcal{U}_{r<r_*}) \oplus H^*(\mathcal{U}_{r>r_*})$$

ILLUSTRATIVE EXAMPLE : ADJOINT HIGGSING OF 7D SUPER-YANG-MILLS

Setup : M-theory on $\mathbb{R}^{6,1} \times \mathbb{C}^2/\mathbb{Z}_N$ $\rightsquigarrow$ 7D SYM w/ $\mathfrak{g} = \mathfrak{su}(N)$

$$\mathrm{SymTFT}_{8D} \supseteq \frac{N}{2\pi}\int B_2 \cup \delta C_5$$

$$G_4 = B_2 \cup t_2$$

$S^3/\mathbb{Z}_N \supseteq$ Hopf Circle

$\longrightarrow$

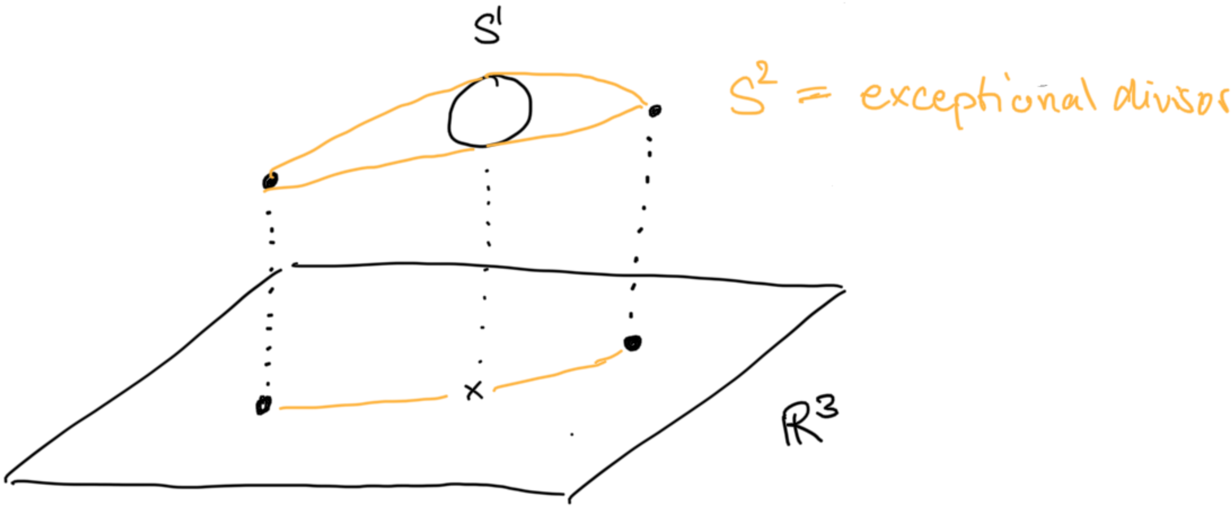
$\check{\mathbb{Z}}_N$ Center Sym. $\longleftrightarrow$ B_2, C_5

Adjoint Higgsing $\Longleftrightarrow$ Partial Resolution

$$X \longrightarrow \tilde{X}$$

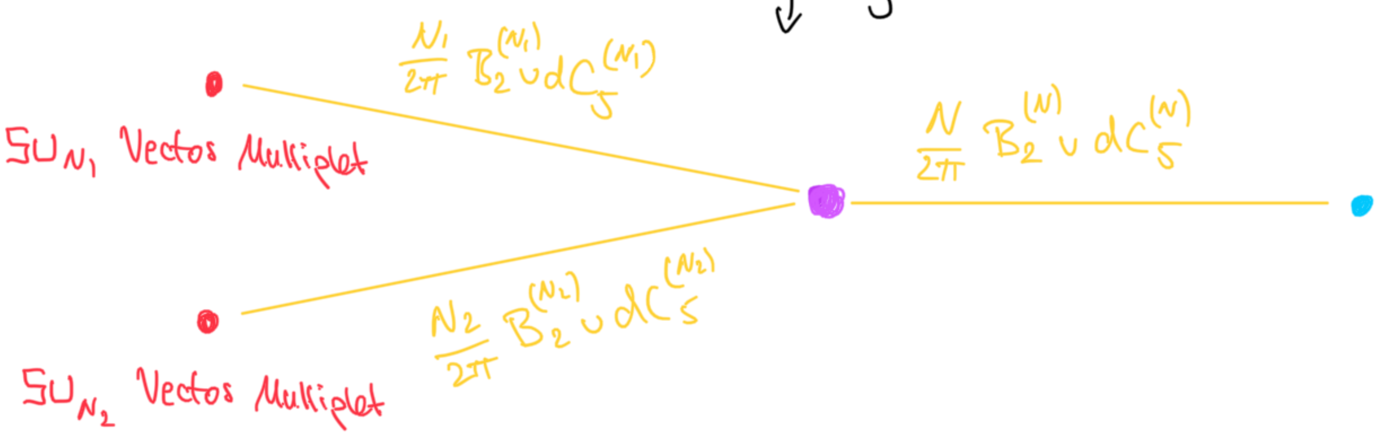
$$xy = z^N \longrightarrow xy = (z - N_2 t)^{N_1}(z + N_1 t)^{N_2}$$

$$SU_N \longrightarrow SU_{N_1} \oplus SU_{N_2} \oplus U_1 \qquad N = N_1 + N_2$$



Deriving The SymTree :

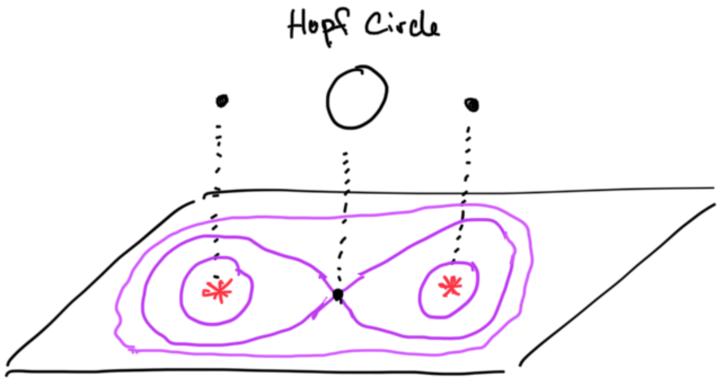
$\downarrow$ SymTree



ARE THERE DEGREES OF FREEDOM AT JUNCTION ?

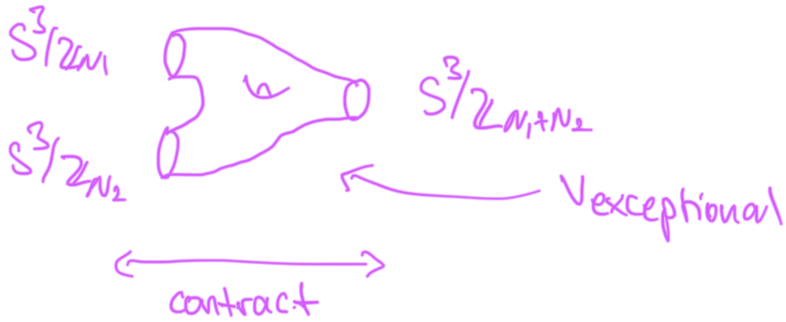
WHAT BOUNDARY CONDITIONS DOES JUNCTION IMPOSE ON THE THREE SYMTFTS ?

Compactifying SUGRA on radial shells : exceptional shell $\longleftrightarrow$ Junction



Exceptional shell : $\quad \mathcal{U}_{\text{exceptional}} = S^3/\mathbb{Z}_{N_1} \cup_{\text{Hopf Circle}} S^3/\mathbb{Z}_{N_2}$

Deformation Retraction of



$\Longrightarrow \quad \mathcal{U}_{\text{exceptional}} = V_{\text{exceptional}}\big|_{\text{retract}}$

The geometric avatars of the SymTFT fields $B_2^{(N_1)}, B_2^{(N_2)}, B_2^{(N_1+N_2)}$ are the Hopf Circles of $S^3/\mathbb{Z}_{N_1}, S^3/\mathbb{Z}_{N_2}, S^3/\mathbb{Z}_{N_1+N_2}$ respectively.

There are embeddings :

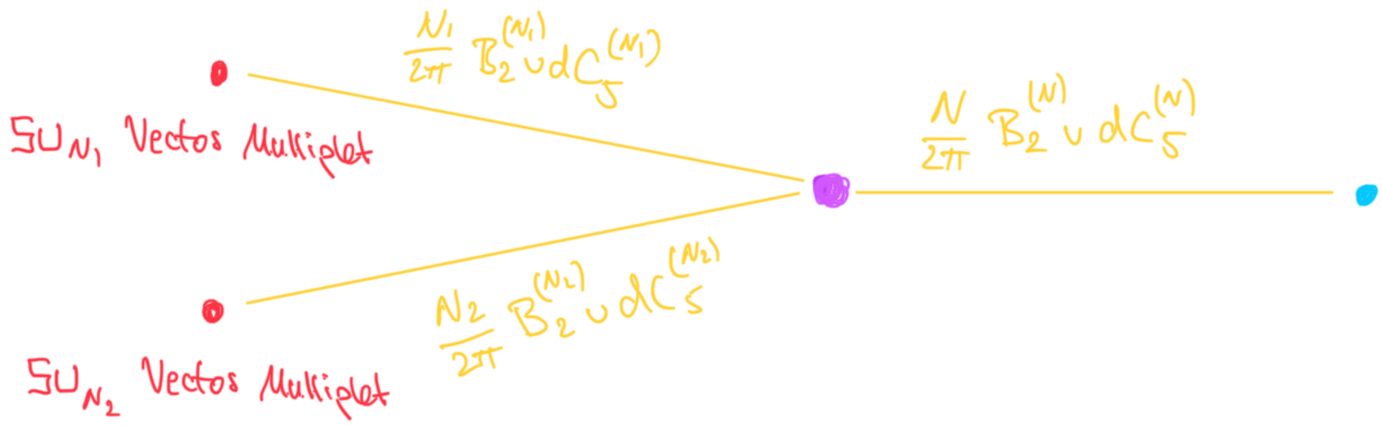
$$S^3/\mathbb{Z}_{N_1} \hookrightarrow \mathcal{U}_{\text{exceptional}} \hookleftarrow S^3/\mathbb{Z}_{N_1+N_2}, \qquad S^3/\mathbb{Z}_{N_2} \hookrightarrow \mathcal{U}_{\text{exceptional}}$$

$\Longrightarrow$ We can compare Hopf cycles in $\mathcal{U}_{\text{exceptional}}$

$\Longrightarrow$ Gluing conditions for $B_2^{(N_1)}, B_2^{(N_2)}, B_2^{(N_1+N_2)}$ at Junction

$$\frac{N_1}{g} B_2^{(N_1)}\Big|_{\text{Junction}} = \frac{N_1}{g} B_2^{(N_1)}\Big|_{\text{Junction}} = \frac{N}{g} B_2^{(N)}\Big|_{\text{Junction}}$$

Topological Boundary Conditions!

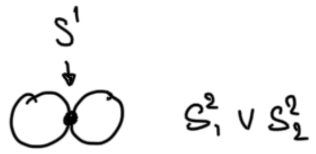


Boundary Conditions incomplete: $\dfrac{\mathbb{Z}_{N_1} \oplus \mathbb{Z}_{N_2} \oplus \mathbb{Z}_N}{\mathbb{Z}_g \oplus \mathbb{Z}_g}$

Dynamics at the Junction

(Co)homology for SUGRA Reduction

$$H_n\Big(\underbrace{S^3/\mathbb{Z}_{N_1} \cup_{\text{Hopf Circle}} S^3/\mathbb{Z}_{N_2}}\Big) = \begin{cases} \mathbb{Z} & n=0 \\ \mathbb{Z}_g & n=1 \\ \mathbb{Z} & n=2 \\ \mathbb{Z}^2 & n=3 \end{cases}$$



$$H^n\Big(S^3/\mathbb{Z}_{N_1} \cup_{\text{Hopf Circle}} S^3/\mathbb{Z}_{N_2}\Big) = \begin{cases} \mathbb{Z} & n=0 \\ 0 & n=1 \\ \mathbb{Z} \oplus \mathbb{Z}_g & n=2 \\ \mathbb{Z}^2 & n=3 \end{cases}$$

- $H_1\left(S^3/\mathbb{Z}_{N_1} \cup_{\text{Hopf Circle}} S^3/\mathbb{Z}_{N_2} \right) \cong \mathbb{Z}_g$

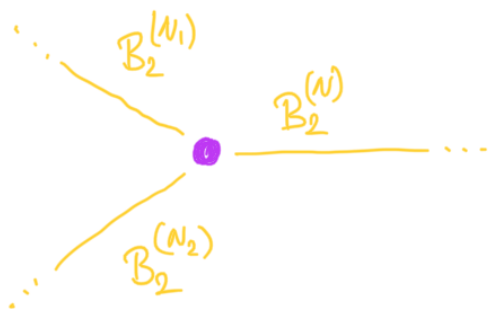
  Reflecting the gluing cond. above being an eq. in $\mathbb{Z}_g$, and that $1 \quad \mathbb{Z}_g$ remains fluctuating

- $H_2\left(S^3/\mathbb{Z}_{N_1} \cup_{\text{Hopf Circle}} S^3/\mathbb{Z}_{N_2} \right) \cong \mathbb{Z}$

  Generated by $L = \mathrm{lcm}(N_1, N_2)$ copies of the exceptional curve

  $\Longrightarrow$ $U(1)$ Vectormultiplet localized at Junction

Center perspective on Junction



Dynamics on •

$\Longrightarrow$ $B_2^{(N_1)}, B_2^{(N_2)}, B_2^{(N)}$ admit interpretation as background fields for symmetries of •

computation: consider $\mathbb{Z}_L \subseteq U(1)_J$

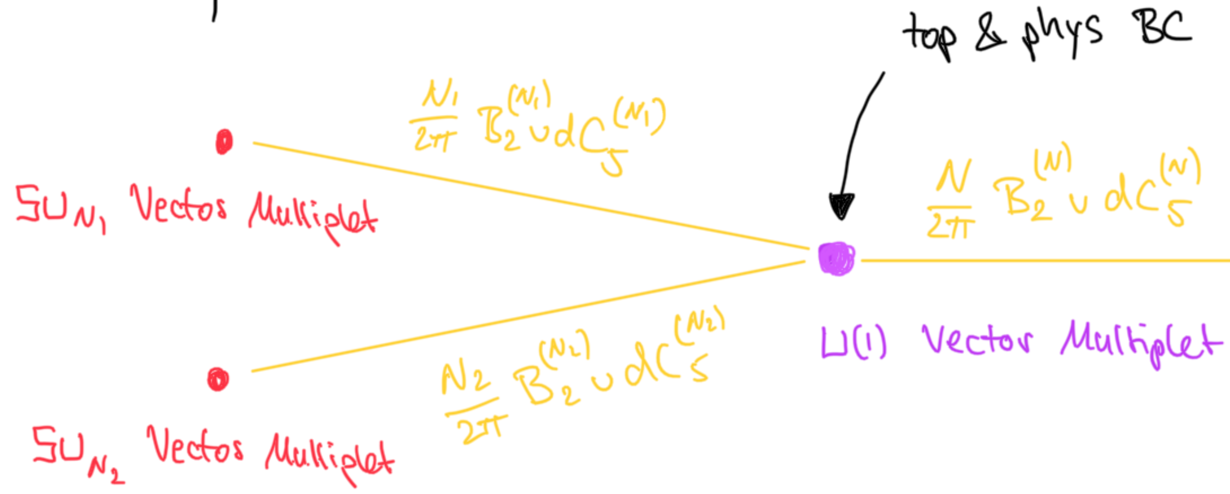
$\hookrightarrow$ Background $B_2^{(L, U(1))}$

$\rightsquigarrow$ $B_2^{(N_1)}, B_2^{(N_2)}, B_2^{(N)}$ give backgrounds for $U(1)_J$ via

$$B_2^{(N_1)} = \frac{N_2}{g} B_2^{(L,U(1))} \qquad B_2^{(N_2)} = \frac{N_1}{g} B_2^{(L,U(1))} \qquad g\, B_2^{(N)} = L\, B_2^{(L,U(1))}$$

Physical Boundary conditions!

Summary so far :



Higgsing Data:

$$SU_N \longrightarrow SU_{N_1} \oplus SU_{N_2} \oplus U_1 \qquad\qquad N = N_1 + N_2$$

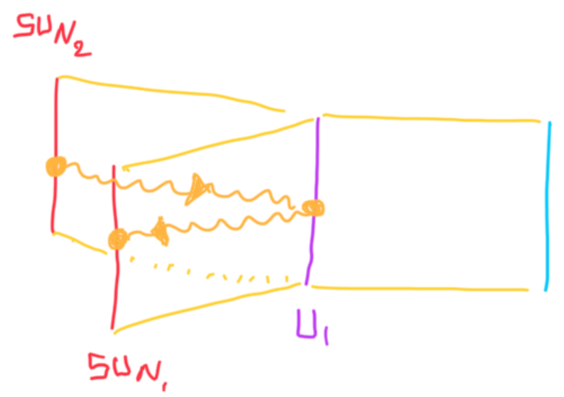
$$\mathrm{Ad}\, SU_N \longrightarrow \mathrm{Ad}\left[SU_{N_1} \oplus SU_{N_2} \oplus U_1\right]$$

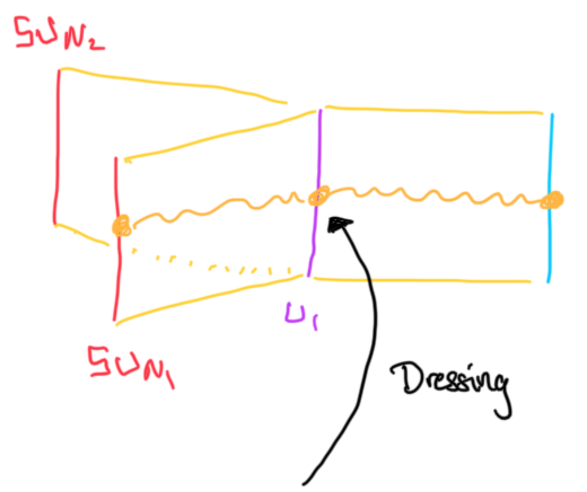
$$\oplus\ (N_1, \bar{N}_2)_{N/g} \oplus (\bar{N}_1, N_2)_{-\frac{N}{g}} \qquad g = \gcd(N_1, N_2)$$

$$N \longrightarrow (N_1, 1)_{N_2/g} \oplus (1, N_2)_{-N_1/g}$$

SymTree is a good description in the limit in which the bifundamentals $(N_1, \bar{N}_2)_{N/g} \oplus (\bar{N}_1, N_2)_{-\frac{N}{g}}$ are heavy and treated as defects :

$$\langle |\vec{\Phi}| \rangle \gg \Lambda$$





$(N_1, 1) \otimes (1, \bar{N}_2) \otimes (1,1)_{N/g}$ $\qquad$ $(N_1, 1) \otimes (1,1)_{N_2/g}$

Comment : • Geometric prescription does not require

Moduli deforming X to a cone,
independent of existence of an unhiggsed theory

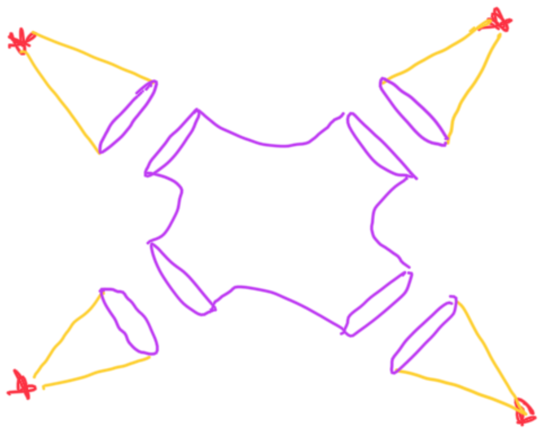
No-Global-Symmetries

Setup : M-theory on compact $X \longrightarrow$ Supergravity Theory S_X

Example : $X = T^4/\mathbb{Z}_2$

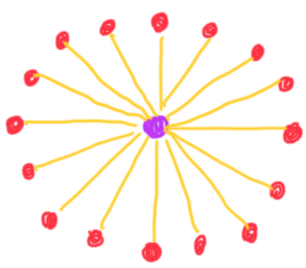
$\downarrow$

16 A_1 singularities $\longrightarrow$ 16 7D $SU(2)$ SYM theories



$$\underbrace{X = X^{loc} \cup X^0}$$

SymTree



Summary : • SymTrees consist of a collection of SymTFTs and interfaces between these

• SymTrees organize Symmetries

Omissions : • Examples without Moduli space flow

• Applications to large N averaging