Generalized Symmetries & Gravity

WIP w/ Cvetič, Heckman, Torres

See also: 2203.10102, 2209.03343, 2212.09743, 2304.0330
2305.09665 w/ " & Acharya, Del Zotto, Yu, Zhang

Motivation:
- Generalized Global Symmetries are ubiquotous in QFTs, but often rely on educated Lagrangian gauge theory constructions.
- Cobordism conjecture / No global symmetries

Data: Topological Symmetry Ops. σ + Fusion

$\circlearrowright$

Non-dynamical Defect Ops $\mathcal{D}$

Example: 4D $U(1)$ Maxwell theory

Flux Ops.: $\mathcal{U}^{(e)}_{\alpha}(\Sigma) = \exp\left(i\alpha \int_{\Sigma} {*F}\right)$

$$\mathcal{U}^{(m)}_{\alpha}(\Sigma) = \exp\left(i\alpha \int_{\Sigma} F\right)$$

$\circlearrowright$

Defect Ops.: Wilson & 't Hooft lines

Question 1: If ~~a~~ QFT w/ GGS admits an embedding into string / M-th: How do $\sigma, \mathcal{D}$ lift?

Question 2: If the local string / M-th model embeds into a global model: What happens to GGS?

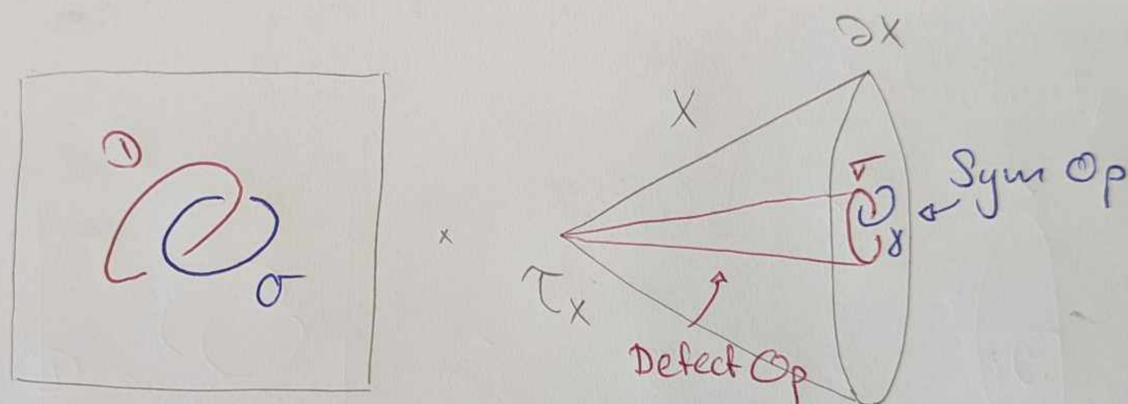
① Geometric Engineering – Local Models

IIA / IIB / M-theory

Non-compact geometry X $\rightarrow$ Relative theory $\mathcal{T}_X$

$X = \mathbb{C}^2/\Gamma,\ \mathbb{C}^3/\Gamma,$ Elliptic CY3, G_2-Mfd., ... (∂X smooth)

Defect Ops.: Branes wrapped on non-cpct relative cycles
Sym. Ops.: (Flux-) Branes wrapped on $\underbrace{\text{asymptotic}}_{\text{"at infinity"}}$ cycles



Defect Group (p-Branes): $\mathbb{D} = \bigoplus_m \mathbb{D}^{(m)}$, $\mathbb{D}^{(m)} \cong \bigoplus_{p-k=m-1} \dfrac{H_k(X,\partial X)}{H_k(X)}$

Discrete Sym.: link
σ is torsional
γ wrapped by EM-dual q-brane,

Continuous Sym.:
σ is free
γ " Flux (q+1)-brane
intersect

Remarks:

- Non-Lagrangian characterization for $\mathcal{D}$'s & $\mathcal{O}$'s
- Streamlined prescription + natural generalizations
- Computationally useful, draws on anomaly theory developed in string theory

Eg.: D-branes

$$\mathcal{O}(M) = \int DA_1 \exp\left[2\pi i \int_{M\times\gamma} \mathcal{L}^{Dp}_{top}\right]$$

$$S^{Dp}_{top} = 2\pi i \int_{\mathcal{M}=M\times\gamma} \exp(\underbrace{F_2}_{dA_1} - B_2)\sqrt{\frac{\hat{A}(T\mathcal{M})}{\hat{A}(N\mathcal{M})}} \underset{odd/even}{\oplus} C$$

Note: Gauge field A_1 is path-integrated over

$\Rightarrow$ World volume TFT_M / $\Rightarrow$ Non-inv. fusion

② Geometric Engineering – Global Models

M / IIA / IIB

compact geometry $X \longrightarrow$ SUGRA theory $\mathcal{S}_X$

Singularities $\longrightarrow$ localized degrees of freedom $\mathcal{T} \subset \mathcal{S}_X$

$\longrightarrow$ local model $X^{\text{loc}} \subset X$ s.t.

$$\mathcal{T} \equiv \mathcal{T}_{X^{\text{loc}}}$$

SUGRA completion : $\mathcal{T}_{X^{\text{loc}}} \hookrightarrow \mathcal{S}_X$

$\Rightarrow$ symmetries gauge / break

Geometry : $X^{\text{loc}} \hookrightarrow X$

- Defect Op. supports compactify
  - $\rightarrow$ Extra massive matter
- Sym. Op. supports trivialize
  - $\rightarrow$ Gauging / trivialization of sym.

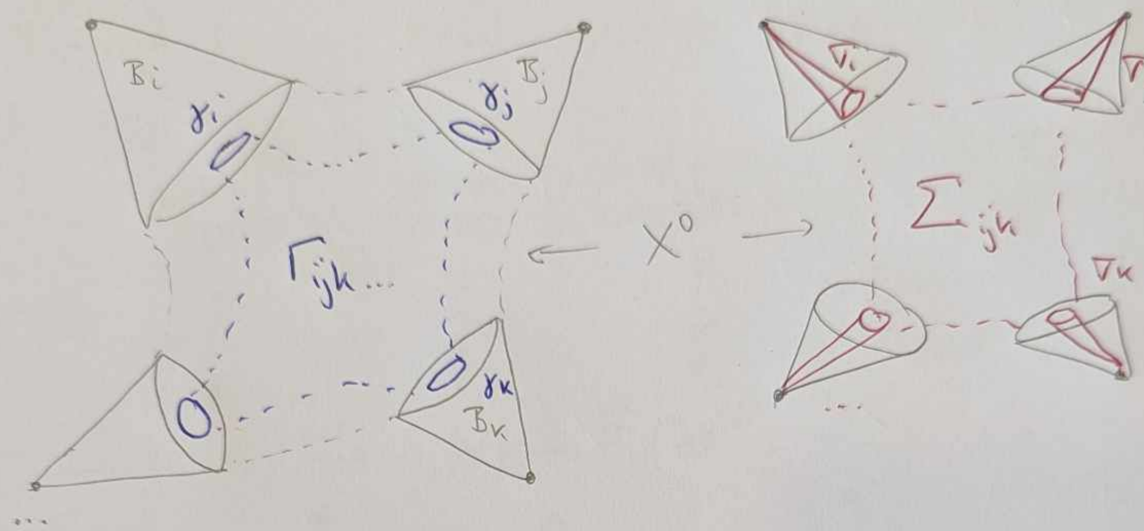
Quantified via Mayer-Vietoris LES :

$$\longrightarrow H_n(X) \longrightarrow H_{n-1}(\partial X^{\text{loc}}) \longrightarrow H_{n-1}(X^{\circ}) \oplus H_{n-1}(X^{\text{loc}}) \longrightarrow \dots$$

w/ covering $X = X^{\text{loc}} \cup X^{\circ}$

$$X^{\circ} \cong X \setminus X^{\text{loc}}$$

Singularities of X have multiple connected components $\longrightarrow$ $X^{loc} = \coprod_i B_i$



③ Example: K3 Surfaces

X singular K3 $\longrightarrow$ ADE singularities at $\{x_i\}$

M-theory on X $\longrightarrow$ 7D SUGRA $\mathcal{S}_X$

w/ local 7D SYM sectors

w/ Lie Algebra $\mathfrak{g}_i$

$\Rightarrow$ <u>Local Model:</u> $X_i^{loc} = \mathbb{C}^2/\Gamma_i$ w/ $\partial X_i^{loc} = S^3/\Gamma_i$

- Defect Ops.: M2-/M5-brane wrapped on Cone(γ_i)
- Sym Ops.: M5-/M2-brane wrapped on γ_i

where $\langle \gamma_i \rangle = \mathrm{Tor}\, H_1(\partial X_i^{loc})$

Overall : $X^{loc} = \coprod_i X_i^{loc}$

w/ defect & sym ops in each component

Extract following exact sequence from MV :

Extra $U(1)$s ③ — Massive Matter ② — Local Model Defect / Sym Ops ① — Emergent / Broken Symmetries ④

$$0 \to H_2(X^\circ) \xrightarrow{j_2} H_2(X) \xrightarrow{\partial_2} H_1(\partial X^{loc}) \xrightarrow{\iota_1} H_1(X^\circ) \to 0$$

$U(1)$ Charge Normalization — Gauging / Trivializing Symmetries ⑤ — Bulk comparison ⑥

Important Duality chain :

$$H_1(X^\circ) \overset{PL}{=} H^3(X^\circ, \partial X^\circ) \overset{EX}{=} H^3(X, \{x_i\}) \overset{RH}{=} H^3(X)$$

UCT : $\mathrm{Tor}\, H_1(X^\circ) = \mathrm{Tor}\, H_2(X)^\vee$ ← Pontryagin Dual

Perfect Pairing

No Global Symmetries $\longleftrightarrow$ Exactness of Mayer-Vietoris

$X^{loc} \hookrightarrow X$ specifies field theory manipulations gauging / breaking all symmetries.

$$0 \to \frac{\text{Free } H_2(X)}{\text{Im } j_2} \oplus \text{Tor } H_2(X) \to \bigoplus_i H_1(\partial X_i^{loc}) \to H_1(X^\circ) \to 0$$

Explicitly: Kummer $T^4/\mathbb{Z}_2$

$$0 \to \mathbb{Z}^6 \xrightarrow{\cdot 2} \mathbb{Z}^6 \oplus \mathbb{Z}_2^5 \to \mathbb{Z}_2^{16} \to \mathbb{Z}_2^5 \to 0$$

$$\Rightarrow \quad G_{SUGRA} = \frac{\left(SU(2)^{16}/\mathbb{Z}_2^5\right) \times U(1)^6}{\mathbb{Z}_2^6}$$

Example : M-theory on $T^6/\mathbb{Z}_3$

$$X^{loc} = 27 \times \mathbb{C}^3/\mathbb{Z}_3$$

$$\mathcal{T}_{X^{loc}} = E_0^{27} \quad (\text{Seiberg SCFT 5D})$$

$$\begin{array}{ccccccccc} 0 \to & H_4(X^\circ) & \to & H_4(X) & \to & H_3(\partial X^{loc}) & \to & \mathrm{Tor}\, H_3(X^\circ) & \to 0 \\ & \mathbb{Z}^9 & & \mathbb{Z}^9 \oplus \mathbb{Z}_3^4 & & \mathbb{Z}_3^{27} & & \mathbb{Z}_3^{17} & \end{array}$$

$$\begin{array}{ccccccccc} 0 \to & H_2(X^\circ) & \to & H_2(X) & \to & H_1(\partial X^{loc}) & \to & \mathrm{Tor}\, H_1(X^\circ) & \to 0 \\ & \mathbb{Z}^9 & & \mathbb{Z}^9 \oplus \mathbb{Z}_3^{17} & & \mathbb{Z}_3^{27} & & \mathbb{Z}_3^4 & \end{array}$$

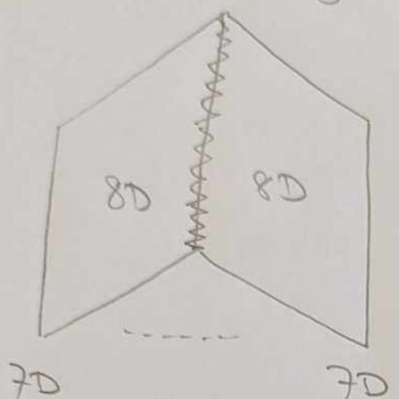
$$G_{SUGRA} = \frac{\mathbb{Z}_3^{27}/\mathbb{Z}_3^4 \times U(1)^9}{\mathbb{Z}_3^6}$$

④ IR Limits

$G_N \to 0 \Rightarrow$ projects onto $\overset{\text{local}}{\text{field theory}}$ sector
+ top. interactions

$\Rightarrow$ Book of symmetry TFTs

(K3)



Pages : local physics X^{loc}

Spine : bulk " X^{0}

$\Rightarrow$ Non-invertible symmetries

(CY3) 5D CS-terms $\sim \frac{k_{IJK}}{24\pi^2} \int A_I \wedge F_J \wedge F_K$

$$\Rightarrow \quad d \star F_I = \sum_{JK}' \frac{k_{IJK}}{8\pi^2} F_J \wedge F_K$$