The Branes Behind Duality Defects

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Motivation

- Generalized global symmetries in QFTs $\Leftrightarrow$ topological operators
  [Gaiotto, Kapustin, Seiberg, Willett, 2014], Introduction $\rightarrow$ Yesterday's Talk by Ibrahima Bah

$$\mathcal{O}\ \text{Symmetry Operators} \quad \circlearrowright \quad \text{Defect Operators}\ \mathcal{D}$$

- Question, If the QFT admits an embedding into string/M-theory:

How do $\mathcal{O}, \mathcal{D}$ lift to string/M-theory?

Punchline:

Branes wrapped on non-compact cycles realize $\mathcal{O}, \mathcal{D}$

[García Etxebarria, 2022], [Apruzzi, Bah, Bonetti, Schafer-Nameki, 2022], [Heckman, MH, Torres, Zhang, 2022]

Application: Duality Defects via 7-branes

[Heckman, MH, Torres, Yu, Zhang, 2022]

Motivation

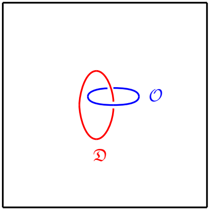
Defects and Symmetry Operators

- Geometric Engineering: IIA/IIB/M theory on $X \Rightarrow$ QFT $\mathcal{T}_X$
- Defect [Del Zotto, Heckman, Park, Rudelius, 2015], [Morrison, Schäfer-Nameki, Willet, 2020], [Albertini, Del Zotto, García Etxebarria, Hosseini, 2020] and Symmetry Operators from p-branes on

$$\mathbb{D} = \oplus_m \mathbb{D}^{(m)}\,, \qquad \mathbb{D}^{(m)} \cong \bigoplus_{p-k=m-1} H_k(X,\partial X)/H_k(X)$$

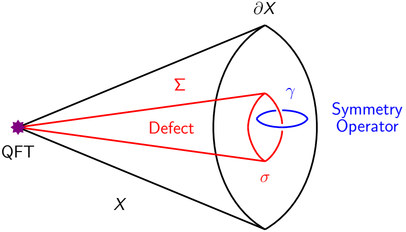
$$\mathbb{O} = \oplus_n \mathbb{O}^{(n)}\,, \qquad \mathbb{O}^{(n)} \cong \bigoplus_{p-k=n-1} H_k(\partial X)$$

- We can sketch the setup as [Heckman, MH, Torres, Zhang, 2022]



Space-time

$\times$



Topological Symmetry Operators

- Symmetry Operators:

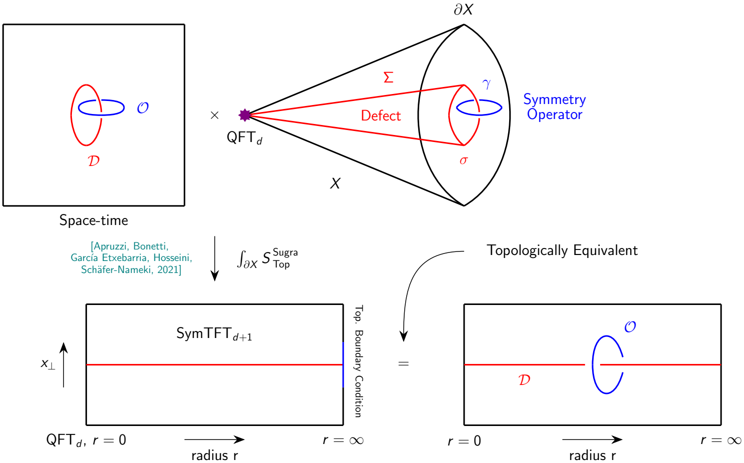
$$\mathcal{O}(M) = \int DA_1 \exp\left(2\pi i \int_{M\times\gamma} \mathcal{L}^{\mathrm{Dp}}_{\mathrm{top}}\right)$$

$$\mathcal{S}^{Dp}_{\mathrm{top}} = 2\pi i \int_{\mathcal{M}=M\times\gamma} \exp(\mathcal{F}_2)\sqrt{\frac{\widehat{A}(T\mathcal{M})}{\widehat{A}(N\mathcal{M})}} \bigoplus_{\mathrm{odd/even}} C_q$$

with $\mathcal{F}_2 = F_2 - B_2$ [Douglas, 1995], [Minasian, Moore, 1997], ...

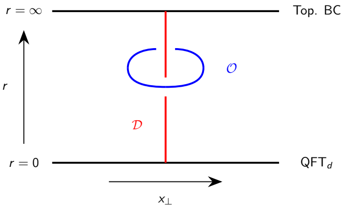
- Gauge field A_1, with field strength F_2, is path-integrated over
$\Rightarrow$ Worldvolume TFT$_M$, Non-invertible Fusion Rules, ...

Symmetry TFT of QFT $\mathcal{T}_X$



In summary:

- Defects: Branes on non-compact, 'radial' cycles
- Symmetry Operators: Branes on asymptotic cycles at $r = \infty$
- $\Rightarrow$ Symmetry TFT: Defects stretch between boundaries ($r = 0, \infty$), while symmetry operators are at finite radius



Duality Defects of $\mathcal{N} = 4$ SYM Theory

- N D3 Branes $\Rightarrow \mathcal{N} = 4$ SYM theory $|\mathcal{T}_N\rangle$
- $\mathcal{L}^{\text{IIB}}_{\text{top}} \propto F_5 \wedge B_2 \wedge dC_2 \Rightarrow$ Symmetry TFT with action [Witten, 1998]

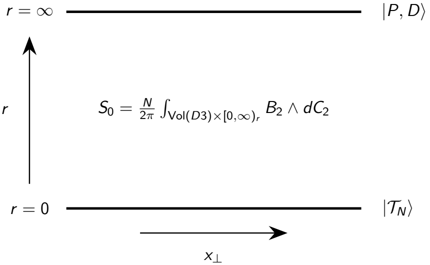
$$S_0 = \frac{N}{2\pi} \int_{\text{Vol}(N \times D3) \times [0,\infty)_r} B_2 \wedge dC_2$$

- Polarization P specify 'position and momentum basis' [Gaiotto, Moore, Neitzke, 2010], [Seiberg, Taylor, 2011], [Aharony, Seiberg, Tachikawa, 2013], [Freed, Telemann, 2014]
- Mixed Neumann/Dirichlet boundary conditions $|P, D\rangle$, s.t.:

$$\langle P, D | \mathcal{T}_N \rangle = Z_{\mathcal{T}_{N,P}}(D)$$

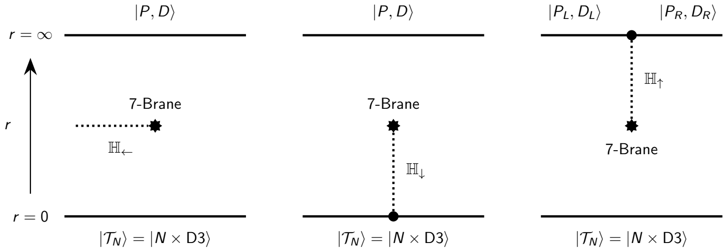
[Kaidi, Zafrir, Zheng, 2022], [Kaidi, Ohmori, Zheng, 2022]

- Polarization $P \Rightarrow$ permissible defects $\mathcal{D}$ and symmetry operators $\mathcal{O}$



7-Brane Insertions

- Wrap a 7-brane on the asymptotic S^5 linking the D3s
- Physically distinct choices of branch cut $\mathbb{H}$:



- Crossing branch cut: $\tau \to \tau'$
- 5D action deformed to $S = S_0 + S_1 + S_2$ (Index = codimension)

Case $\mathbb{H}_{\leftarrow}$

- Defects constructed from (p,q) strings [Bergman, Hirano, 2022], [Antinucci, Benini, Copetti, Galati, Rizi, 2022]

  $\Rightarrow$ 7-Brane monodromy ρ acts on defects

  $\Rightarrow$ Half-Spaces with different 'effective' Polarization

  $\Rightarrow$ Half-Space gauging construction for duality defects [Choi, Cordova, Hsin, Lam, Shao, 2021 & 2022]

- Branch cut supports a counter term

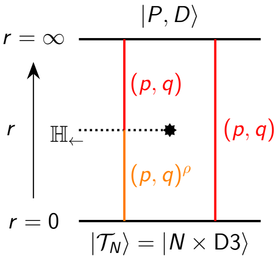
$$S_1 = S_{\text{cut}} = \frac{2\pi i}{N}\int_{\mathbb{H}_{\leftarrow}} \frac{\mathcal{P}(B_2^{\rho})}{2}$$

  with monodromy eigenvector B_2^{ρ}.

- Colliding the branch cut with $r=\infty$ we find

$$|P,D\rangle \to |P',D'\rangle$$

  with possible counter term stacked.



Case $\mathbb{H}_{\downarrow}$

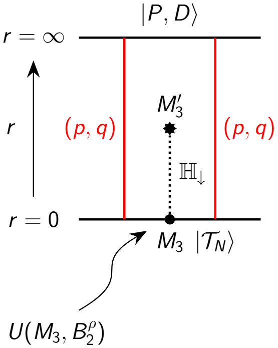
- Branch cut $\mathbb{H}_{\downarrow}$ runs radially
  $\Rightarrow$ Branch cut intersects D3 worldvolume
  $\Rightarrow$ Polarizations are the same in half-spaces
  $\Rightarrow$ Operator $U(M_3, B_2^{\rho})$ at D3 $\cap\, \mathbb{H}_{\downarrow}$
- Anomaly cancellation: The combination

$$U(M_3, B_2^{\rho}) \exp\left(\frac{2\pi i}{N}\int_{\mathbb{H}_{\downarrow}} \frac{\mathcal{P}(B_2^{\rho})}{2}\right) U_{7\text{-brane}}(M_3', B_2^{\rho})$$

  is invariant under background transformations of B_2^{ρ}
- KW-like duality defect [Kaidi, Ohmori, Zheng, 2022] :
  Contract slab $5D \to 4D \Rightarrow$ Branch cut $\mathbb{H}_{\downarrow}$ is contracted, $M_3 = M_3'$,

$$U(M_3, B_2^{\rho}) \otimes U_{7\text{-brane}}(M_3, B_2^{\rho})$$

  in theory $\mathcal{T}_{N,P}(D)$



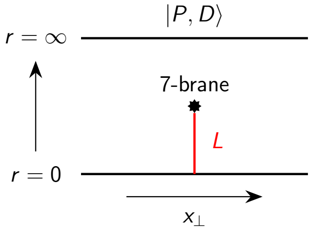
7-Brane Theory

- 7-brane on S^5 $\Rightarrow$ 3D TFT$_{\text{7-brane}}$
- Who is the 3D TFT$_{\text{7-brane}}$? (Hard Question)
- How does the 7-brane theory interact with the D3 stack? (Easy Question) $\Rightarrow$ what are the lines of 3D TFT$_{\text{7-brane}}$?

- F/M-theory duality $\Rightarrow$ Lines L of order K (homology torsion) and spin p (refined self-linking number)
- 3D TFT$_{\text{7-brane}}$ factors [Hsin, Lam, Seiberg, 2018]

$$\mathcal{A}^{K,p}[B_2^\rho] \otimes \mathfrak{T}$$

fully with minimal abelian 3D TFT $\mathcal{A}^{K,p}$ and decoupled 3D TFT $\mathfrak{T}$



Result/Punchline

4D Theory: D3-branes probing Calabi-Yau threefold X

Duality Defect/Interface: 7-brane wrapped on ∂X

Example: $\mathcal{N} = 4$ $\mathfrak{su}(2)$ SYM

- 7-brane of type III*, $\tau = i$, with monodromy

$$\rho = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

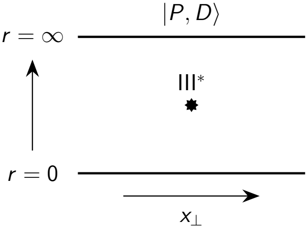
- Possible boundary conditions: [Aharony, Seiberg, Tachikawa, 2013]

$$SU(2)_i\,,\ \ SO(3)_{+,i}\,,\ \ SO(3)_{-,i}$$

where $i = 0, 1$ for possible counter terms $\mathcal{P}(B_P)$,

$$B_{SU(2)} = B_2\,,\ \ B_{SO(3)_+} = C_2\,,\ \ B_{SO(3)_-} = B_2 + C_2$$

with coefficients mod $N = 2$.



Example: $\mathcal{N}=4$ $\mathfrak{su}(2)$ SYM

- Monodromy eigenvector mod 2:

$$B_2^\rho = B_{SO(3)_-} = B_2 + C_2$$

- Lines L have $K=2$ and $p=1$
$\Rightarrow$ 7-brane on S^5 gives the 3D TFT

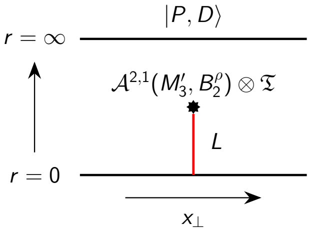
$$\mathcal{A}^{2,1}(M_3', B_2^\rho) \otimes \mathfrak{T}$$

- Mixed anomaly of the $SO(3)_-$ theory [Gaiotto, Kapustin, Seiberg, Willett, 2014], [Cordova, Dumitrescu, 2018]

$$\pi \int_{M_5} A^{(1)} \cup \frac{\mathcal{P}(B_2^\rho)}{2}$$

matching the branch cut term $\propto \mathcal{P}(B_2^\rho)$ on $\mathbb{H}$

- Branch Cut: Radially inwards [Kaidi, Ohmori, Zheng, 2022], horizontal at constant radius [Choi, Cordova, Hsin, Lam, Shao, 2021 & 2022]



Omissions and Outlook

Omissions

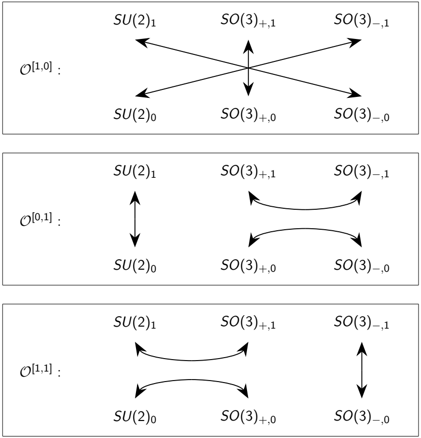
- Extension: D3s probing Calabi-Yau cones X with isolated singularities ($\mathcal{N} = 1$)
  - Symmetry TFT is extended as parametrized by ∂X
  - Construction of duality defects via 7-branes carries over
- 7-brane world volume discussion, $\mathcal{A}^{K,p} \otimes \mathfrak{T}$

Outlook

- 'Branes at Infinity': **Turn the Crank**

Branch Cut Operators

$$
\begin{aligned}
&\langle P_{SU(2)_0}, D| \exp\left(i\pi \int \mathcal{P}(B_2^\rho)/2\right) \\
&= \sum_d \langle P_{SU(2)_0}, D \,|\, P_{SO(3)_{-,0}}, d\rangle \langle P_{SO(3)_{-,0}}, d| \exp\left(i\pi \int \mathcal{P}(d)/2\right) \\
&= \sum_d \langle P_{SU(2)_1}, D \,|\, P_{SO(3)_{-,0}}, d\rangle \langle P_{SO(3)_{-,0}}, d| \exp\left(i\pi \int \mathcal{P}(D)/2 + \mathcal{P}(d)/2\right) \\
&= \sum_d \langle P_{SO(3)_{-,0}}, d| \exp\left(i\pi \int \mathcal{P}(D)/2 + D\cup d + \mathcal{P}(d)/2\right) \\
&= \sum_d \langle P_{SO(3)_{-,1}}, d| \exp\left(i\pi \int D\cup d\right) \exp\left(i\pi \int \mathcal{P}(D)/2\right) \\
&= \langle P_{SO(3)_{+,1}}, D| \exp\left(i\pi \int \mathcal{P}(D)/2\right) \\
&= \langle P_{SO(3)_{+,0}}, D|
\end{aligned}
$$


$\mathcal{O}^{[1,0]}$:
$SU(2)_1$
$SO(3)_{+,1}$
$SO(3)_{-,1}$
$SU(2)_0$
$SO(3)_{+,0}$
$SO(3)_{-,0}$
$\mathcal{O}^{[0,1]}$:
$SU(2)_1$
$SO(3)_{+,1}$
$SO(3)_{-,1}$
$SU(2)_0$
$SO(3)_{+,0}$
$SO(3)_{-,0}$
$\mathcal{O}^{[1,1]}$:
$SU(2)_1$
$SO(3)_{+,1}$
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$SU(2)_0$
$SO(3)_{+,0}$
$SO(3)_{-,0}$