Probing Higgs Bundles for Local G_2-Manifolds

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Overview

Introduction and Motivation

- M-theory on local (i.e. non-compact) G_2-manifolds engineers minimally supersymmetric gauge theories in 4d.

  [Acharya, 2000], [Witten, 2001], [Atiyah, Witten, 2003], [Pantev, Wijnholt, 2009], [Braun, Cizel, H, Schafer-Nameki, 2018], [Barbosa, Cvetič, Heckman, Lawrie, Torres, Zoccarato, 2019], [Cvetič, Heckman, Rochais, Torres, Zoccarato, 2020], [H, 2020], [Acharya, Kinsella, Svanes, 2020], [Acharya, Najjar, Foscolo, Svanes, 2020]

- The 4d gauge theories describe the localized degrees of freedom in compact G_2-manifolds.

  [Acharya, 1998], [Halverson, Morrison, 2015], [Guio, Jockers, Klemm, Yeh, 2017], [Braun, Schafer-Nameki, 2017], [Braun, Del Zotto, 2017], [Braun, 2019], [Xu, 2020]

- F-theory methods relying on Higgs bundles and their spectral covers can be applied to study the physics of local G_2-manifolds. [Beasley, Heckman, Vafa, 2009], [Hayashi, Kawano, Tatar, Watari, 2009], [Marsano, Saulina, Schafer-Nameki, 2009], [Marsano, Saulina, Schafer-Nameki, 2010], [Blumenhagen, Grimm, Jurke, Weigand, 2010], [Donagi, Wijnholt 2011], [Donagi, Wijnholt 2014]
- Supersymmetric sigma models probing the geometries give insight into non-perturbative effects and indices. [Alvarez-Gaumé, Witten, 1981], [Witten, 1982], [Pantev, Wijnholt, 2009], [Braun, Cizel, H, Schafer-Nameki, 2018], [H, 2020]

Question: What determines the 4d physics engineered by local G_2-manifolds in M-theory?

ALE Fibered, Local G_2-Manifolds

Geometric data

$$\text{Local } G_2\text{-Manifold} : \quad \widetilde{\mathbb{C}^2/\Gamma_{\text{ADE}}} \hookrightarrow X_7 \to M_3$$

$$\text{Fibral 2-Spheres} : \quad \sigma_I \in H_2(\widetilde{\mathbb{C}^2/\Gamma_{\text{ADE}}}, \mathbb{R})$$

$$\text{Hyperkähler Triplet} : \quad (\omega_1, \omega_2, \omega_3) \in H^2(\widetilde{\mathbb{C}^2/\Gamma_{\text{ADE}}}, \mathbb{R})$$

The Higgs field collects the Kähler periods

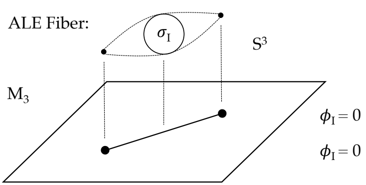
$$\text{Higgs field} : \quad \phi_I = \left(\int_{\sigma_I} \omega_i \right) dx^i \in \Omega^1(M_3)$$

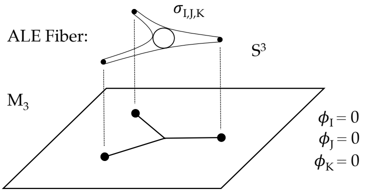
where $I = 1, \ldots, \text{rank}\, \mathfrak{g}_{\text{ADE}}$.

Singularities and Supersymmetric 3-cycles

Singularity Enhancement at $x \in M_3$: $\phi_I(x) = 0$ (isolated)

The vanishing cycles trace out 3-spheres:





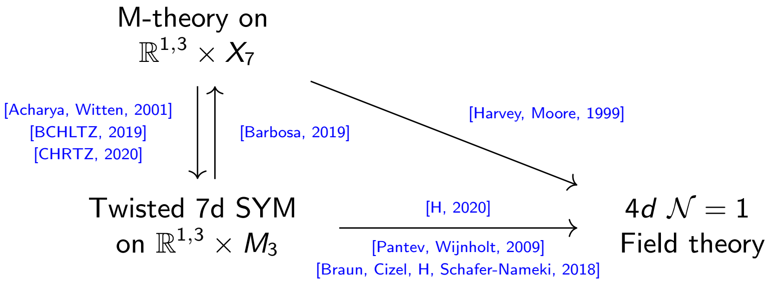
Questions

What is the global structure of the network of supersymmetric 3-spheres and the M2-brane instantons?

What do the M2-branes wrapped on the supersymmetric 3-spheres descend to in an effective 7d field theory description?

How can their contribution to the 4d superpotential be computed from the 7d field theory description?

Previous Work



Effective 7d Physics

M-theory on the local G_2-manifold X_7 gives a

Partially twisted 7d SYM on $\mathbb{R}^{1,3} \times M_3$
with gauge group G_{ADE} .

Complex bosonic 1-form on M_3 : $\varphi = \phi + iA \in \Omega^1(M_3, \mathfrak{g}_{\text{ADE}})$

Supersymmetric vacua are solutions to the Hitchin system:

$$i(F_A)_{ij} + [\phi_i, \phi_j] = 0 \,, \qquad (d_A\phi)_{ij} = 0 \,, \qquad * d_A * \phi = 0$$

Zero modes along M_3 are determined by

$$H = \frac{1}{2}\left\{Q, Q^\dagger\right\}, \qquad Q = d + [\varphi, \cdot\,]$$

and counted by the cohomologies

$$H^*_Q(M_3, \mathfrak{g}_{\text{ADE}}) .$$

Alternatively, consider perturbative zero modes

$$\chi_a \in \Omega^*(M_3, \mathfrak{g}_{\text{ADE}}) \qquad \leftrightarrow \qquad \text{Codimension 7 Singularity}$$

The zero modes are recovered from

$$\text{Mass Matrix}: \qquad M_{ab} = \int_{M_3} \langle \chi_b, Q\chi_a \rangle$$

$$\text{Yukawa Couplings}: \qquad Y_{abc} = \int_{M_3} \langle \chi_c, [\chi_a, \chi_b] \rangle .$$

Colored $\mathcal{N} = 2$ SUSY Quantum Mechanics

Bosonic Coordinates on M_3 : $x^i\,, \quad i = 1,2,3$

Fermions in $x^*(TM_3)$: $\psi^i\,, \quad i = 1,2,3$

Color fermions in $x^*(\mathrm{ad}\, G_{\mathrm{ADE}})$: $\lambda^\alpha\,, \quad \alpha = 1,\dots,\dim \mathfrak{g}_{\mathrm{ADE}}$

Lagrangian :

$$\mathcal{L} = \frac{1}{2}\dot{x}^i\dot{x}_i - \frac{1}{2}\phi^i_\lambda\phi_{\lambda,i} + i\bar{\psi}^i\nabla_\tau\psi_i + i\bar{\lambda}^\alpha D_\tau\lambda_\alpha - \left(D_{(i}\phi_{j)}\right)_\lambda \bar{\psi}^i\psi^j + i\left(F_{ij}\right)_\lambda \bar{\psi}^i\psi^j + \zeta\left(\bar{\lambda}^\alpha\lambda_\alpha - 1\right)$$

Hilbertspace : $\mathcal{H}_{\mathrm{phys.}} = \Lambda(M_3, \mathfrak{g}_{\mathrm{ADE}})$

Supercharge : $Q = d + [\varphi, \cdot\,]$

$$\text{Bosonic Coordinates on } M_3 : \quad x^i\,, \quad i = 1,2,3$$

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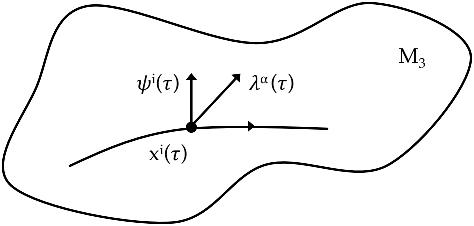
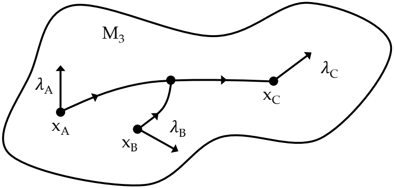


Supercharge : $Q = \bar{\psi}_i \left(\dot{x}^i - \phi^i_\lambda \right)$

Perturbative Groundstates : $(x, \lambda) \in M_3 \times \mathfrak{g}_{\text{ADE}}$ with $\phi_\lambda(x) = 0$

1/2-BPS instantons are piecewise solutions to the flow equations

Flow line instanton : $\dot{x}^i - \phi^i_\lambda = \dot{x}^i - i c^\alpha{}_{\beta\gamma} \phi^i_\alpha \bar{\lambda}^\beta \lambda^\gamma = D_\tau \lambda^\alpha = 0 \, .$



Colored instantons $\longleftrightarrow$ Euclidean M2-brane instantons

Abelian Higgs Backgrounds (split)

Abelian solutions to the BPS equations: $d_A = d\,, \;\; \phi = \phi_I H^I\,.$

The 1-form Cartan components ϕ_I are harmonic up to sources

$$d\phi_I = 0\,, \qquad * d * \phi_I = \rho_I \quad \rightarrow \quad \phi_I = df_I\,, \qquad \Delta f_I = \rho_I\,,$$

which are supported on codimension ≥ 2 subloci in M_3.

Root $\alpha \in \mathfrak{g}_{\text{ADE}}$ $\leftrightarrow$ Witten SQM into M_3 with supercharge $Q^{(\alpha)} = d + \alpha^I df_I$

Yukawa Couplings

$$
\begin{aligned}
Y_{abc} &= \int_{M_3} \langle \chi_c^{(\gamma)}, [\chi_a^{(\alpha)}, \chi_b^{(\beta)}] \rangle \\
&= \int_{M_3} d^3 x_0 \int DxD\psi D\bar{\psi} \, \exp \left[i \left(S^{(\alpha)}[x_a, \psi_a, \bar{\psi}_a] + S^{(\beta)}[x_b, \psi_b, \bar{\psi}_b] + S^{(\gamma)}[x_c, \psi_c, \bar{\psi}_c] \right) \right] \\
&= \sum_{\Gamma_{abc}} (\pm)_{\Gamma_{abc}} \exp \left(- \int_{\Gamma_a} \alpha^I \phi_I - \int_{\Gamma_b} \beta^I \phi_I + \int_{\Gamma_c} \gamma^I \phi_I \right) \\
&= \sum_{\Gamma_{abc}} (\pm)_{\Gamma_{abc}} \exp \left[- \mathrm{Vol} \left(S^3_{\Gamma_{abc}} \right) \right] \qquad \rightarrow \qquad \text{[Harvey, Moore, 1999]}
\end{aligned}
$$

$$
\chi_r^{(\alpha)} = \lim_{T \to -\infty(1+i\delta)} \frac{e^{-iHT} \Psi_r \, e^{iHT}}{e^{-iE_{0,r}T} \langle \chi_r | \Psi_r^{(\alpha)} \rangle} \equiv \Psi_r^{(\alpha)} \Big|_{-\infty} , \qquad 0 < \delta \ll 1 ,
$$

$$
DxD\psi D\bar{\psi} = \prod_{\substack{-\infty < \tau < 0 \\ x_{a,-\infty} = x_a \\ x_{b,-\infty} = x_b}} d \left\{ x, \psi, \bar{\psi} \right\}_{a,\tau} \, d \left\{ x, \psi, \bar{\psi} \right\}_{b,\tau} \prod_{\substack{0 < \tau < \infty \\ x_{c,\infty} = x_c}} d \left\{ x, \psi, \bar{\psi} \right\}_{c,\tau}
$$

Effective 4d Theory (split)

We find:

1. Mathematics fixing effective 4d physics is Fukaya's Morse theory with multiple Morse functions, [Fukaya, 1997]
2. Chiral and conjugate chiral multiplets in 4d counted by

$$H^1(M_3, \partial_-^{(\alpha)} M_3)\,,\ H^2(M_3, \partial_-^{(\alpha)} M_3)$$

   respectively. [Pantev, Wijnholt, 2009],[Braun, Cizel, H, Schafer-Nameki, 2018]
3. Yukawa couplings given by a cup-product on $H^*(M_3, \partial_-^{(\alpha)} M_3)$

Abelian Higgs Backgrounds (non-split)

Abelian solutions to the BPS equations: $d_A = d\,, \quad \phi = \mathrm{diag}(\Lambda_K)\,.$

With eigenvalue 1-forms Λ harmonic up to source terms

$$d\Lambda_K = *j_K\,, \qquad *\,d*\Lambda_K = \rho_K\,,$$

which are supported on codimension ≥ 1 subloci in M_3. (TCS)

Two classes of solutions distinguished by their spectral cover:

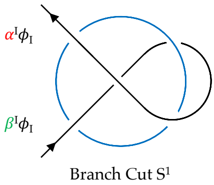
$$\mathcal{C} = \{(x, \Lambda_K(x))\,|\,x \in M_3\} \subset T^*M_3 \quad \rightarrow \quad \begin{cases} \Lambda_K \text{ globally defined} \\ \Lambda_K \text{ connected by branch cuts} \end{cases}$$

Colored SQM for Non-split Spectral Covers

The eigenvalue 1-forms of the Higgs field are interchanged along paths linking the branch locus

$$\text{Monodromy Action} : \quad \phi \to g\phi g^{-1}$$
$$\text{Color Mixing} : \quad E^\alpha \to gE^\alpha g^{-1}$$

Monodromy orbit $[\alpha]$ $\leftrightarrow$ Witten SQM into $\mathcal{C}_k$ with supercharge $Q^{([\alpha])} = d + \phi_{[\alpha]}$



Effective 4d Theory (non-split)

We find:

1. Gauge symmetry determined by stabilizer of ϕ
2. Chiral and conjugate chiral multiplets in 4d counted by

$$H^1_{\mathrm{Nov.}}(\mathcal{C}_k, \phi_{[\alpha]})\,,\;\; H^2_{\mathrm{Nov.}}(\mathcal{C}_k, \phi_{[\alpha]})$$

3. Yukawa couplings given by a cup-product on $H^*_{\mathrm{Nov.}}(\mathcal{C}_k, \phi_{[\alpha]})$

Summary

We introduced a colored SQM probing Higgs bundles of local G_2-manifolds, and establish the correspondence

Euclidean M2 Instantons $\leftrightarrow$ Instantons of Colored SQM

For abelian backgrounds we compute instanton effects giving the

$$\text{Mass Matrix}: \qquad M_{ab} = \int_{M_3} \langle \chi_b, Q\chi_a \rangle$$

$$\text{Yukawa Couplings}: \qquad Y_{abc} = \int_{M_3} \langle \chi_c, [\chi_a, \chi_b] \rangle \ .$$

and determine the effective 4d physics.

Outlook

- Extend on non-split Higgs fields and Novikov cohomologies
- Analyze fluxed T-brane solutions

  [Barbosa, Cvetič, Heckman, Lawrie, Torres, Zoccarato, 2019]
- Understand the M-theory origin of the colored SQM

  [Harvey, Moore, 1999]
- Explore other Higgs bundle vacua

  [Cvetič, Heckman, Rochais, Torres, Zoccarato, 2020]

The End

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