The Branes Behind Topological Symmetry Operators

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Motivation: Generalized Global Symmetries

They are useful:

- Phase and vacua structures in 2d theories ($\rightarrow$ see Lakshya's talk)
- Confinement and deconfinement in 4d theories
- Neutrino masses from generalized symmetry breaking
  [Cordova, Hong, Koren, Ohmori, 2022]
- No global symmetries and cobordism conjecture ($\rightarrow$ see Jonathan's, Markus' talk)

and they are ubiquitous. ($\rightarrow$ see Mirjam's, Yi-Nan's, Paul's, Inaki's, Muyang's talk)

Often rely on educated Lagrangian gauge theory constructions.

Question

Data of generalized global symmetries includes:

- Topological symmetry operators $\mathcal{O}$
- Non-dynamical defect operators $\mathcal{D}$ (Representations)
- Fusion structure (often non-invertible)

Given a theory with generalized global symmetries which admits an embedding into string theory: How do $\mathcal{O}, \mathcal{D}$ lift?

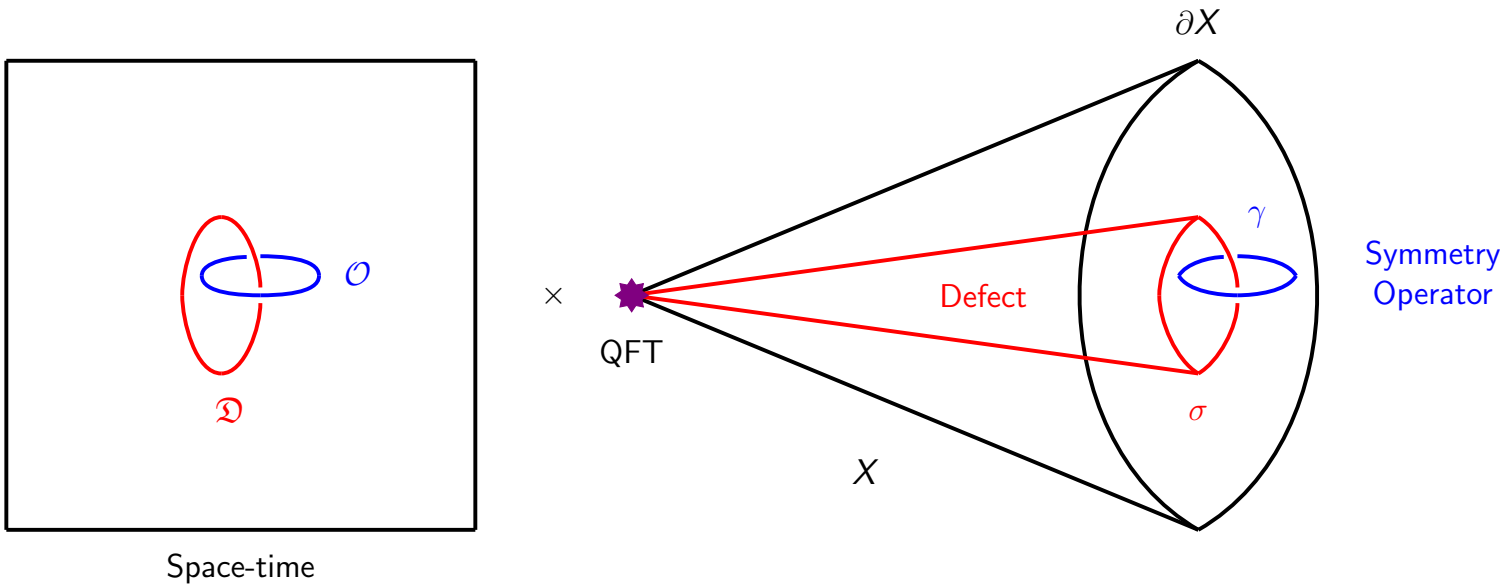
Generalized Global Symmetry in String Theory

Example, Geometric Engineering:

IIA/IIB/M theory on non-compact X $\Rightarrow$ Theory $\mathcal{T}_X$

Then, defect ops and symmetry ops can be constructed from (flux)branes.

[Del Zotto, Heckman, Park, Rudelius, 2015], [Morrison, Schäfer-Nameki, Willet, 2020], [Albertini, Del Zotto, García Etxebarria, Hosseini, 2020], [García Etxebarria, 2022], Apruzzi, Bah, Bonetti, Schäfer-Nameki, 2022], [Heckman, MH, Torres, Zhang, 2022], and more



Philosophy applies more broadly.

D-branes and Symmetry Operators

Symmetry operators from topological worldvolume terms:

$$\mathcal{O}(M) = \int DA_1 \exp\left(2\pi i \int_{M\times\gamma} \mathcal{L}^{\mathrm{Dp}}_{\mathrm{top}}\right)$$

with world volume gauge field A_1 and Wess-Zumino action:

$$\mathcal{S}^{Dp}_{\mathrm{top}} = 2\pi i \int_{\mathcal{M}=M\times\gamma} \exp(\mathcal{F}_2)\sqrt{\frac{\widehat{A}(T\mathcal{M})}{\widehat{A}(N\mathcal{M})}} \bigoplus_{\mathrm{odd/even}} C_q$$

where $\mathcal{F}_2 = F_2 - B_2$ [Douglas, 1995], [Minasian, Moore, 1997], ...

Important: Gauge field A_1 is path-integrated over

$\Rightarrow$ Worldvolume TFT_M, Non-invertible Fusion Rules, ...

Examples

- Non-invertible symmetries in 6D SCFTs [Heckman, MH, Torres, Zhang, 2022]

  IIB String Theory: $X = \mathbb{C}^2/\Gamma$ & $X =$ NHC, D3-brane on 1-cycles of ∂X

- Duality defects [Heckman, MH, Torres, Yu, Zhang, 2022]

  IIB String Theory: $N\times$D3 probing $X = \mathbb{C}^3$, 7-brane on $\partial X = S^5$

- Verlinde's metastable monopole [Cvetič, Heckman, MH, Torres, 2023]

- SCFT junctions via G_2 [Acharya, Del Zotto, Heckman, MH, Torres, 2023]

  M-theory: $X =$ G2/Γ, defects via M2-branes on cones

- Charge Conjugation 6D $\mathcal{N} = (2,0)$ [Dierigl, Heckman, Montero, Torres, 2022]

  IIB String Theory: $X = \mathbb{C}^2/\Gamma$, R7-brane on $\partial X = S^3/\Gamma$

- 4D $\mathcal{N}$=4 SYM [Garcìa Etxebarria, 2022]

- 4D $\mathcal{N}$=1 SYM [Apruzzi, Bah, Bonetti, Schäfer-Nameki, 2022]

- S-folds [Etheredge, Garcìa Etxebarria, Heidenreich, Rauch, 2023]

and more...

Teaser: Generalized Symmetries and Gravity

Consider compact singular X (e.g., $T^4/\mathbb{Z}_2$)

M-theory on $X \to$ supergravity theory $\mathcal{S}_X$ (with localized 7d SYM sectors)

Singularity content: A_1^{16}

Define local models and complement:

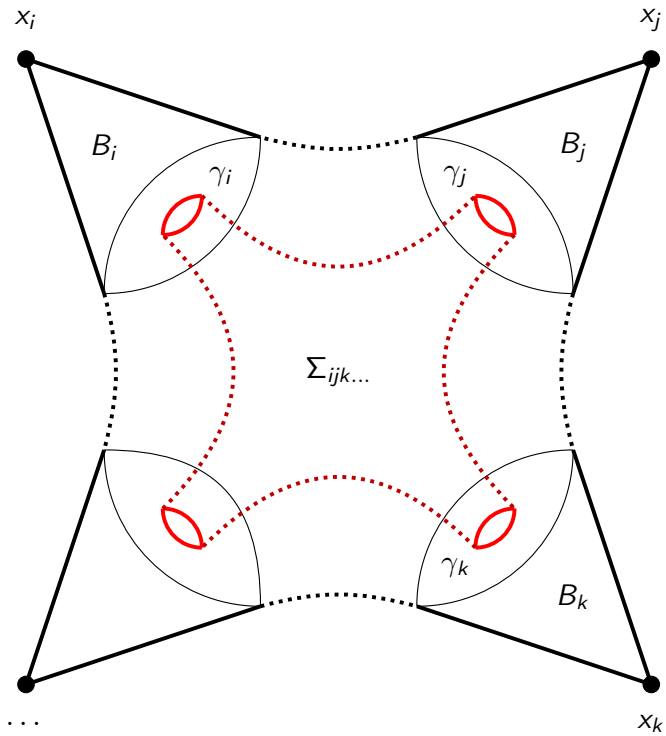
$$X^{\text{loc}} = \cup_i B_i \,, \qquad X^\circ = X \setminus X^{\text{loc}}$$

Cutting and gluing:

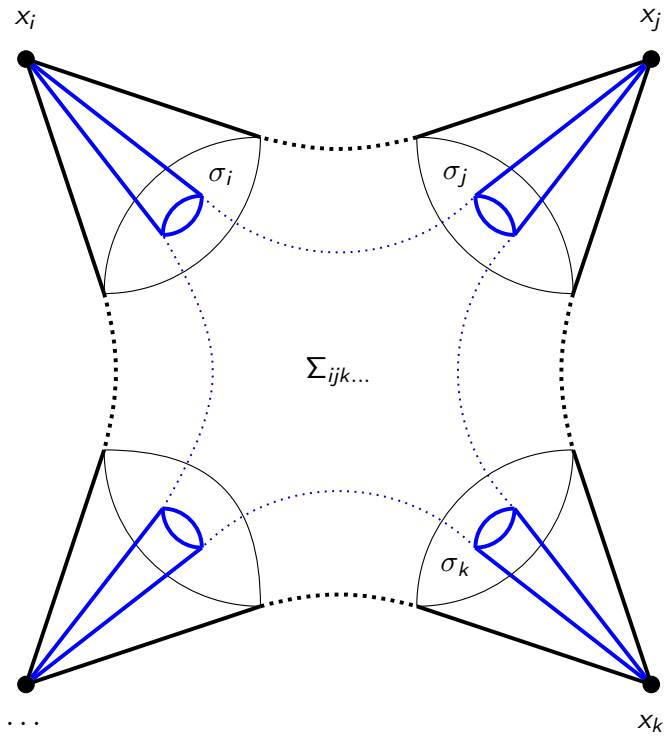
$$0 \;\to\; H_2(X^\circ) \;\xrightarrow{\jmath_2}\; H_2(X) \;\xrightarrow{\partial_2}\; H_1(\partial X^{\text{loc}}) \cong \oplus_i H_1(\partial B_i) \;\xrightarrow{\imath_1}\; H_1(X^\circ) \;\to\; 0$$

$$0 \to H_2(X^\circ) \xrightarrow{\jmath_2} H_2(X) \xrightarrow{\partial_2} H_1(\partial X^{\text{loc}}) \cong \oplus_i H_1(\partial B_i) \xrightarrow{\imath_1} H_1(X^\circ) \to 0$$

Sketch of geometry $X = T^4/\mathbb{Z}_2$:



(1) : Trivialization of Symmetry Operators



(2) : Compactification of Defect Operators

Thank you for your time.