Living on the edge: Interfaces of SCFTs via orbifolds of G_2-spaces (to appear)

w/ Acharya, Del Zotto, Heckman, Torres

① Introduction (Physics)

often perturbative

90s+: "Intersecting Brane Models" ← eg quiver GT

Duality

"Singular Metric Profiles"
(or other singular SUGRA backgrounds)

non-perturbative generalizations

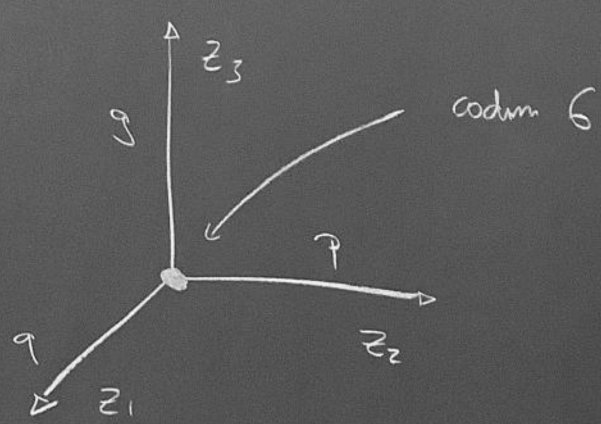
eg 5D/6D SCFTs

Motivating Example: $X^6_{pq} = \mathbb{C}^3/\Gamma$, $\Gamma = \mathbb{Z}_p \times \mathbb{Z}_q$

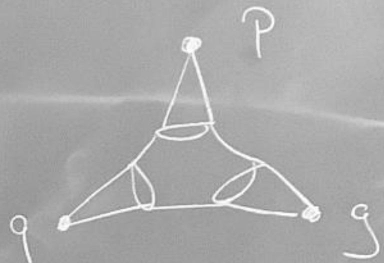
$$(z_1, z_2, z_3) \sim (\omega z_1, \eta z_2, \omega^{-1}\eta^{-1} z_3)$$

w/ $\omega = \exp(2\pi i/p)$, $\eta = \exp(2\pi i/q)$

Fixed pts in $\mathbb{C}^3$	Order	Singularities in $X^6_{p,q}$
$z_3 = z_1 = 0$	p	A_{p-1} @ $\mathbb{C}_{z_2}/\mathbb{Z}_q$
$z_2 = z_3 = 0$	q	A_{q-1} @ $\mathbb{C}_{z_1}/\mathbb{Z}_p$
$z_1 = z_2 = 0$	$g = \gcd(p,q)$	A_{g-1} @ $\mathbb{C}_{z_3}/(\Gamma/\mathbb{Z}_g)$



Asymptotic Bdry : $\partial X^6_{pq} = S^5/\mathbb{Z}_p \times \mathbb{Z}_q$
(flat metric)



$$H_2(\partial X^6_{p,q}) \cong \mathbb{Z}_g$$

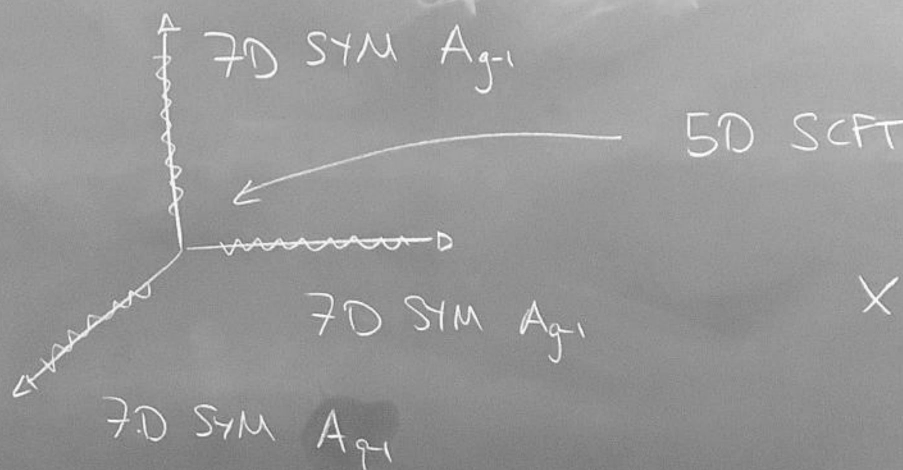
$\Longrightarrow$ Non-compact relative 3-cycle :

$$H_3(X^6_{pq}, \partial X^6_{pq}) \cong \mathbb{Z}_g$$

Toric Smoothing : $\tilde{X}^6_{pq}$

$\hookrightarrow$ Compact 2 & 4 cycles

M-th on $X^6_{pq} \times \mathbb{R}^{1,5}$:



$\times \quad \mathbb{R}^{1,5}$

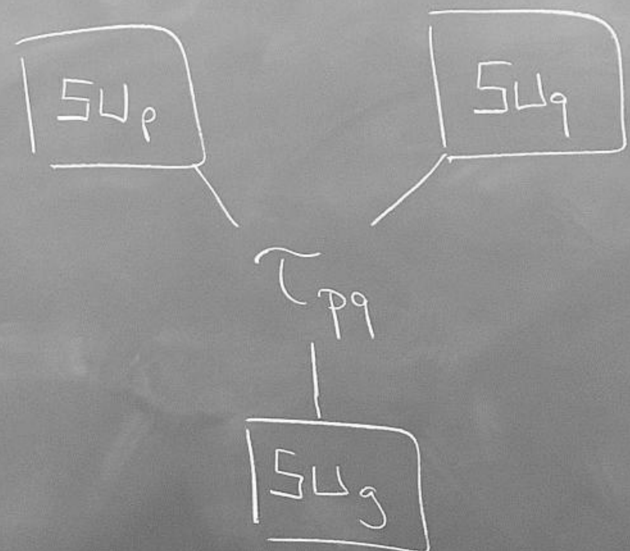
$\Longrightarrow$ 5D Theory at intersection of / along interface between 3 × 7D Theories

RG $\downarrow$ $\quad$ IR free

Genuine 5D SCFT $\mathcal{T}_{pq}$ w/ non-ab flavor sym.

$$F = \mathfrak{su}_p \oplus \mathfrak{su}_q \oplus \mathfrak{su}_g$$

"Quiver Picture":



Non-ab Flavor Group $\quad \mathcal{F} = \dfrac{SU(p)\times SU(q)\times SU(g)}{\mathbb{Z}_p \times \mathbb{Z}_q}$

$$Z_{\mathcal{F}} = \mathbb{Z}_g \cong H_3(X^6_{pq}, \partial X^6_{pq})$$

M2 Branes on non-compact three cycles give local operators charged under $Z_{\mathcal{F}}$

② General Considerations

Consider: $X^d = \text{cone}\left(Y^{d-1}\right)$

$$ds^2_{X^d} = dr^2 + r^2 ds^2_{Y^{d-1}}$$

(non-compact, special holonomy, ...)

$\mathcal{S} = \text{IIA}, \text{IIB}, M \qquad D_{\mathcal{S}} = 10, 10, 11$

" $\mathcal{S}$ on $\mathbb{R}^{1,D-1} \times X^d \longleftrightarrow \mathcal{T}^{(\mathcal{S})}_{X^d} \in QFT_D \,, D = D_{\mathcal{S}} - d$ "

Cases:

- Isolated Singularity: the singularity at the tip of the cone is isolated & of codim d

  $\Rightarrow$ SCFT (maybe trivial) eg $\mathbb{C}^3/\mathbb{Z}_3$

- Non-isolated Singularity: the singularity at the tip of the cone arises at the collision of higher codim singularities.

  $\Rightarrow$ Interface SCFT (eg. motivating example)

  A)

  B)

A) Higher codim singularity is IR free or gapped

$\Longrightarrow$ local d.o.f. supported at the higher codim sing decouples in the IR

(Start of talk)

$\rightsquigarrow$ " genuine SCFT "

B) Higher codim sing supports an SCFT in the IR $\Longrightarrow$ no necessary decoupling

(Rest of talk)

$\rightsquigarrow$ " non-genuine / edge / quasi - SCFT "

(3) 4D $\mathcal{N}=1$ Edgemodes via G_2-cones in M-theory

Recipe: X G_2-cone, $\Gamma \leq \mathrm{Isom}(X)$,

$$H_3(X) = 0$$

$\Rightarrow$ M-th on X/Γ is $\mathcal{N}=1$,

no quantum corrections due to

Euclidean M2-brane instantons

Case Study: $X = \Lambda^2_{ASD}(S^4)$, $S^4 = \mathbb{HP}^1$

$\partial X = \mathbb{CP}^3$, $\mathrm{Isom}(X) = SO(5)$

$\mathbb{CP}^3$: $[z_1, z_2, z_3, z_4] \sim [\lambda z_1, \lambda z_2, \lambda z_3, \lambda z_4]$ $\lambda \in \mathbb{C}^*$

$\mathbb{HP}^1$: $[Q_1, Q_2] \sim [\mu Q_1, \mu Q_2]$ $\mu \in \mathbb{H}^*$ (left)

$\mathbb{CP}^3 \to \mathbb{HP}^1$ $[z_1 : z_2 : z_3 : z_4] \mapsto [z_1 + z_2 j : z_3 + z_4 j]$

Orbifolding Data.

(Right)

$Sp(2)/_{\mathbb{Z}_2}$: $[Q_1 : Q_2] \mapsto [Q_1\lambda_{11} + Q_2\lambda_{21} : Q_1\lambda_{12} + Q_2\lambda_{22}]$

$\dfrac{Sp(1)_{(1)} \times Sp(1)_{(2)}}{\mathbb{Z}_2}$: $[Q_1 : Q_2] \longmapsto [Q_1\lambda_1 : Q_2\lambda_2]$

Affine Patches $\mathbb{HP}^1$: $q_{north} = Q_2^{-1}Q_1 \mapsto \lambda_2^{-1} q_{north} \lambda_1$

$q_{south} = Q_1^{-1}Q_2 \longmapsto \lambda_1^{-1} q_{south} \lambda_2$

Consider $\Gamma \cong \mathbb{Z}_k$

$$\Gamma \leq Sp(1)_{(1)} \times Sp(1)_{(2)} \leq Sp(2)/_{\mathbb{Z}_2} = \mathrm{Isom}(X)$$

$q_{north} = v_1 + v_2 j$ $\qquad$ $q_{south} = v_1' + v_2' j$

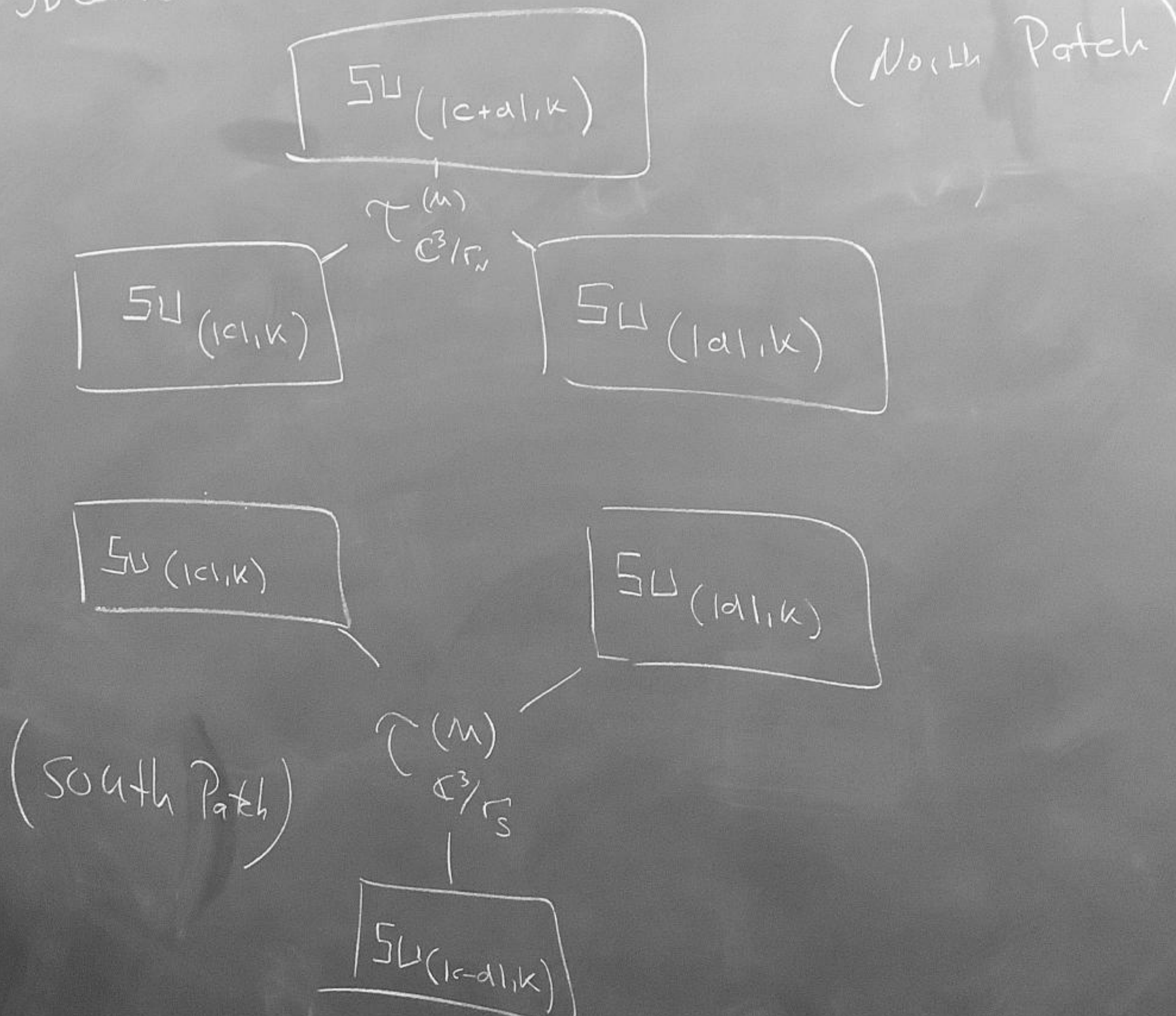
North: $\quad (v_1, v_2, v_3, t) \mapsto (\zeta^{c} v_1, \zeta^{d} v_2, \zeta^{-c-d} v_3, t)$

South: $\quad (v_1', v_2', v_3', t') \mapsto (\zeta^{-c} v_1', \zeta^{d} v_2', \zeta^{c-d} v_3', t')$

$\zeta = \exp(2\pi i/k)$ $\qquad$ $c, d \in \mathbb{Z}$

locally $\mathbb{C}^3/\Gamma \times \mathbb{R}_{t,t'}$

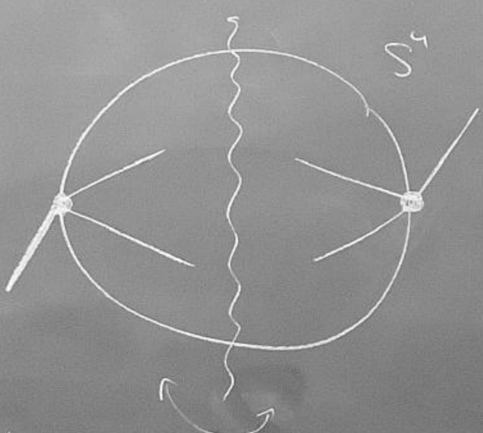
"5D Quiver Picture":



Global Geometry: $\Lambda^2_{ASD}(S^4)/\mathbb{Z}_K$

V_1, V_2, V_1', V_2' compact coords on S^4

V_3, t, V_3', t' non-compact fiber coords



glue $\rightsquigarrow$ compactify $\rightsquigarrow$ gauge

5D SCFTs along

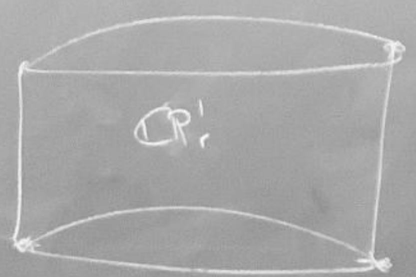
Space-time $\times \mathbb{R}_{t,t'} \times \left\{ \begin{matrix} \text{north} \\ \text{south} \\ \text{pole} \end{matrix} \right\}$

Radial Slice $\mathbb{CP}^3/\Gamma$:

∘ codim 6

| codim 4

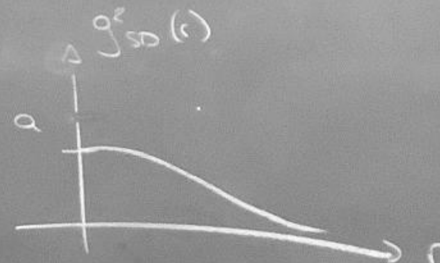
Radial Slice $\mathbb{CP}^3/\Gamma$



codim 6

codim 4

$$\mathrm{Vol}(\mathbb{CP}^1_r) \sim \frac{1}{g^2_{SD}(r)}$$

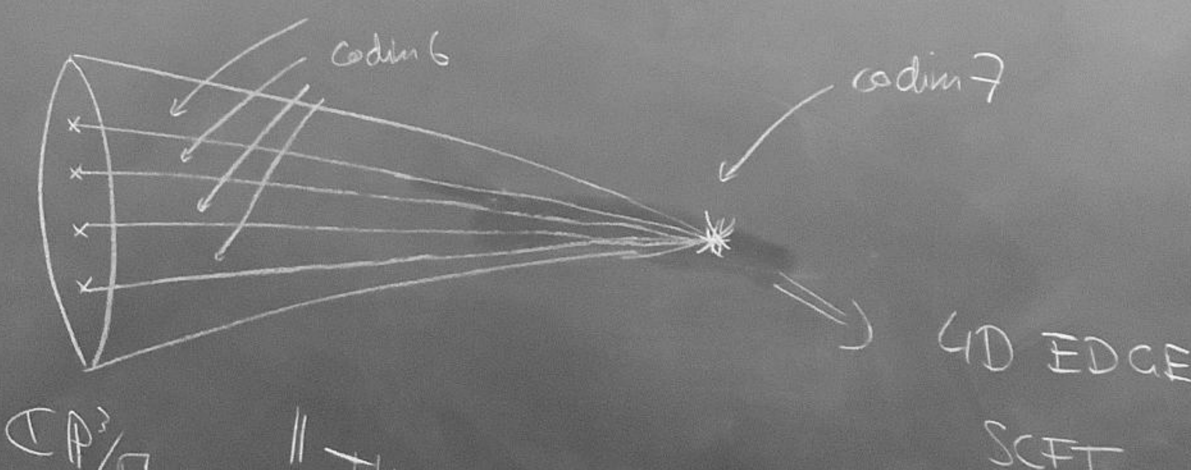


Massive Modes M2 on $H_2(S^4/\Gamma) \cong \mathbb{Z}_k$

Conical Limit : $\mathrm{Vol}(S^4/\Gamma) \to 0$

$$\mathbb{R}_{t,t'} \xrightarrow{\text{break}} \mathbb{R}_{t,t' \geq 0} \cup \mathbb{R}_{t,t' \leq 0}$$

Only drawing codim-6 loci / all gcds = 0 :



"THE QUADRION"

(4) Symmetries from Geometry

5D SCFT supported radially

$\Longrightarrow$ 5D gauge $\oplus$ flavor sym $\longrightarrow$ 4D flavor sym.

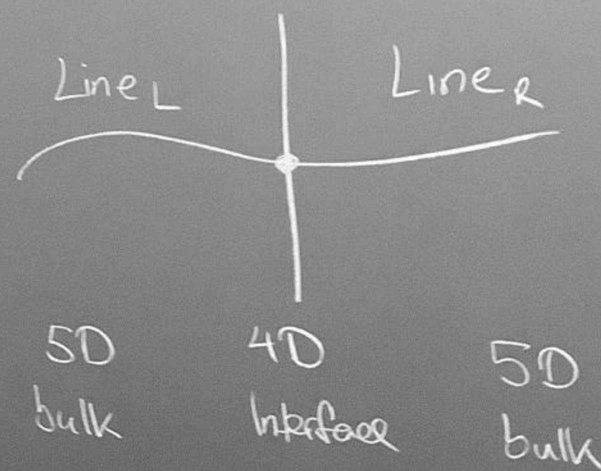
Spontaneous Symmetry Breaking (0-form)

$$\mathrm{Vol}(S^4/\Gamma) = 0 \ : \ \underbrace{su_{(|c-d|,k)} \oplus su_{(|c+d|,k)}}_{\mathfrak{h}} \oplus \underbrace{su^2_{(|c|,k)} \oplus su^2_{(|d|,k)}}_{\mathfrak{g}_+ \oplus \mathfrak{g}_-}$$

$$\mathrm{Vol}(S^4/\Gamma) \neq 0 \qquad \mathfrak{h} \oplus \mathfrak{g}_{diag}$$

Symmetry Inheritance

Space-time :



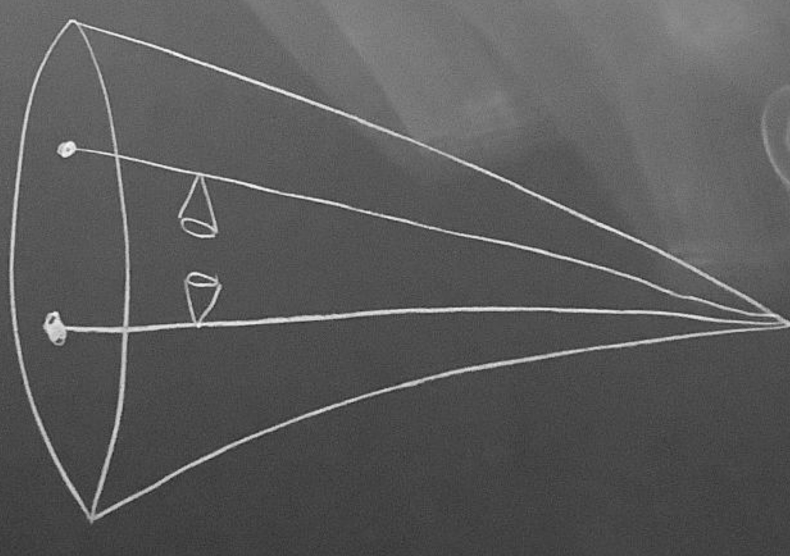
"4D local operator inherited from 5d line"

Generally : q-form symmetry in d dim. inherited from p-form sym in D dim , $d < D$, $q < p$

Internal Geometry : (Defect Perspective)

Recall $\mathbb{D} \sim \bigoplus_{k} \dfrac{H_k(X, \partial X)}{H_k(X)}$

Apply to local models of higher dim Theories



Formalize Σ' singular locus of $\partial X = \mathbb{C}P^3/\Gamma$

$T(\Sigma')$ tube

$\partial X^\circ = \partial X \setminus \Sigma'$

$\Longrightarrow \partial X = \partial X^\circ \cup T(\Sigma')$

MV sequence $\longrightarrow$ boundary maps:

$$\partial_k : H_k(\partial X) \to H_{k-1}(\partial T(\Sigma'))$$

Exact Seq. of Sym.:

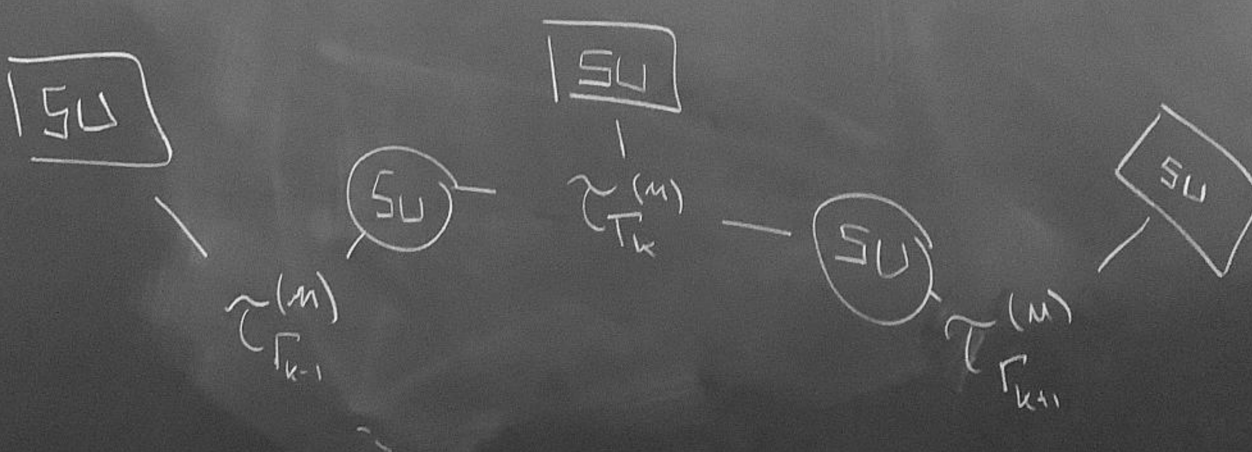
$$0 \to \underbrace{\ker \partial_n}_{\substack{\text{Intrinsic} \\ \text{Defects}}} \to \underbrace{H_n(\partial X)}_{\substack{\cong H_{n+1}(Y, \partial X) \\ \text{all Defects}}} \to \underbrace{\operatorname{Im} \partial_n}_{\substack{\text{Inherited} \\ \text{Defects}}} \to 0$$

(6) Omissions & Generalizations

- ASD Bundles w/ base

$$B = \mathbb{CP}^2, \mathbb{WCP}^2$$

→ 'THE N-ZENES'



- Non-Abelian Quotients

  U1

  SO(5)

$\rightsquigarrow$ compact codim 6 loci

collapse in conical limit