Generalized Global Symmetries and Gravity

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See also: 2203.10102, 2209.03343, 2212.09743, 2304.03300, 2305.09665
with Acharya, Cvetič, Del Zotto, Heckman, Torres, Yu, Zhang

Categorical aspects of symmetries, Nordita

Wednesday August 23rd, 2023

Motivation: Generalized Global Symmetries

Generalized Global Symmetries (GGS) are useful

- Generalized Landau paradigm
- Selection rules, anomalies, RG constraints
- Pheno: Neutrino mass generation [Cordova, Hong, Koren, Ohmori, 2022]

and ubiquitous.

However, their absence is equally interesting:

- No global symmetries in quantum gravity $\rightarrow$ new supergravity defects

  [Banks, Dixon, 1988], [Banks, Seiberg, 2011], [Harlow Ooguri, 2021], [McNamara, Vafa, 2019], [Montero, Vafa, 2020]

  [Debray, Dierigl, Heckman, Montero, 2023], and more

The Problem

M/IIA/IIB on compact singular X $\rightarrow$ supergravity theory $\mathcal{S}_X$

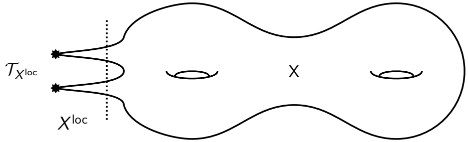
Singularities $\rightarrow$ localized degrees of freedom

$\rightarrow$ QFT sector $\mathcal{T} \subset \mathcal{S}_X$

$\rightarrow$ Local model $X^{\text{loc}} \subset X$ and $\mathcal{T} \equiv \mathcal{T}_{X^{\text{loc}}}$

Questions:

1. How to characterize GGS of $\mathcal{T}_{X^{\text{loc}}}$ via local model X^{loc}?
2. Determine gauging/breaking of GGS when completing $\mathcal{T}_{X^{\text{loc}}}$ to $\mathcal{S}_X$?



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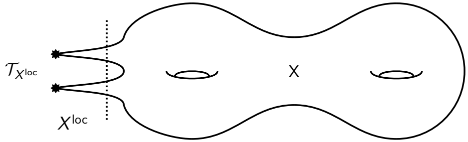


QFTs and Local Models

Data:

- Topological symmetry operators $\mathcal{O}$ [García Etxebarria, 2022], [Apruzzi, Bah, Bonetti, Schäfer-Nameki, 2022], [Heckman, Hubner, Torres, Zhang, 2022], [Cvetič, Heckman, Hubner, Torres, 2022], [Bah, Leung, Waddleton, 2023]
- Non-dynamical defect operators $\mathcal{D}$ (Representations) [Del Zotto, Heckman, Park, Rudelius, 2015], [Morrison, Schäfer-Nameki, Willet, 2020], [Albertini, Del Zotto, García Etxebarria, Hosseini, 2020], [Bhardwaj, Schäfer-Nameki, 2020], [Gukov, Hsin, Pei, 2020]
- Fusion higher Category [Bashmakov, Del Zotto, Hasan, Kaidi, 2022], [Heckman, Hubner, Torres, Yu, Zhang, 2022], [Etheredge, Garcia Etxebarria, Heidenreich, Rauch, 2023], [Bah, Leung, Waddleton, 2023], [Apruzzi, Bonetti, Gould, Schäfer-Nameki, 2023]

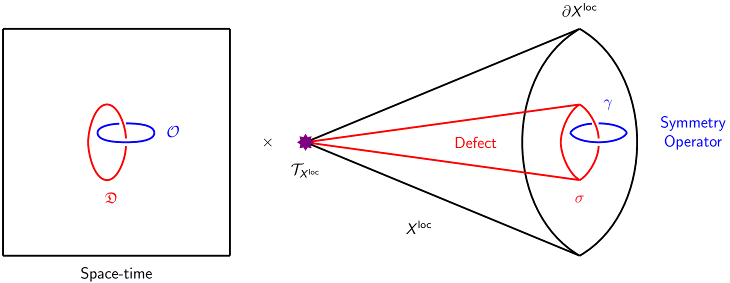
[Roumpedakis, Seifnashri, Shao, 2022], [Choi, Cordova, Hsin, Lam, Shao, 2022], [Andriot, Carqueville, Cribiori, 2022], [Bhardwaj, Bottini, Schäfer-Nameki, Tiwari, 2022], [Gukov, Koroteev, Nawata, Pei, Saberi, 2022], [Bartsch, Bullimore, Ferrari, Pearson, 2022], [Freed, Moore,Teleman, 2022], [Antinucci, Benini, Copetti, Galati, Rizi, 2022], [Dierigl, Heckman, Montero, Torres, 2023], [Bhardwaj, Schäfer-Nameki, 2023], [Bartsch, Bullimore, Grigoletto, 2023], and many more

Given a theory with GGS admitting an embedding into string theory, here via methods of geometric engineering, we ask:

How do $\mathcal{O}, \mathcal{D}$ lift?

Generalized Global Symmetry in String Theory

Defect operators and symmetry operators can be constructed from (flux)branes.



Philosophy applies more broadly, beyond geometric engineering.

Generalized Global Symmetry in String Theory

Defect operators from branes wrapped on relative homology quotients:

$$\mathbb{D}^{(k)} \cong \frac{H_k(X^{\text{loc}}, \partial X^{\text{loc}})}{H_k(X^{\text{loc}})} \cong H_{k-1}(\partial X^{\text{loc}})|_{\text{triv.}} \qquad \text{(previous slide: } \sigma\text{)}$$

Symmetry operators from (flux-)branes wrapped on asymptotic cycles:

$$\mathbb{O}^{(\ell)} \cong H_\ell(\partial X^{\text{loc}}) \qquad \text{(previous slide: } \gamma\text{)}$$

Two cases for Symmetry operators from branes:

- γ is torsional $\rightarrow$ wrap membrane
- γ is free $\rightarrow$ wrap fluxbrane

Note: Branes support worldvolume TFT $\rightarrow$ Non-invertible Fusion Rules

Back to Global Models

Fate of defect operators and symmetry operators when X^{loc} is completed to X?

- Defect operator supports compactify $\rightarrow$ Extra massive matter
- Symmetry operator supports are identified $\rightarrow$ Symmetries trivialize

Quantified via Mayer-Vietoris long exact sequence: [Mayer, 1929], [Vietoris, 1930]

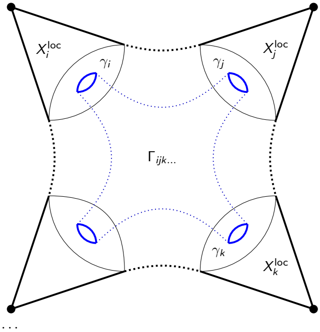
$$\dots \xrightarrow{\jmath_n} H_n(X) \xrightarrow{\partial_n} H_{n-1}(\partial X^{\text{loc}}) \xrightarrow{\imath_{n-1}} H_{n-1}(X^\circ) \oplus H_{n-1}(X^{\text{loc}}) \rightarrow \dots$$

with respect to the covering:

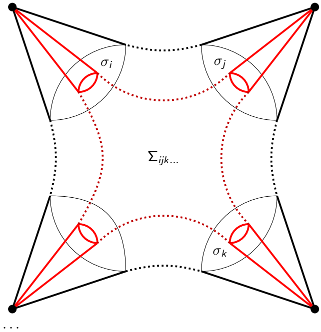
$$X = X^{\text{loc}} \cup X^\circ , \qquad X^\circ = X \setminus X^{\text{loc}}$$

Singularities of X have multiple connected components $\rightarrow$ $X^{\text{loc}} = \coprod_i X_i^{\text{loc}}$

Sketch of geometric interactions between local model components:



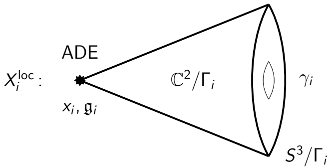
(1) : Symmetry Operators



(2) : Defect Operators

Example: K3 Surfaces

Let X be a singular K3 surface $\rightarrow$ 7D SUGRA $\mathcal{S}_X$ with 7D SYM sectors



Defect Operators: M2-/M5-brane wrapped on Cone(γ_i)

Symmetry Operators: M5-/M2-brane wrapped on γ_i

Overall, the total local model is:

$$X^{\text{loc}} = \coprod_i X_i^{\text{loc}}$$

with a collection of defect and symmetry operators for each component.

Example: K3 Surfaces

From Mayer-Vietoris obtain exact subsequence:

$$0 \;\rightarrow\; H_2(X^\circ) \;\rightarrow\; H_2(X) \;\rightarrow\; H_1(\partial X^{\mathrm{loc}}) \;\rightarrow\; H_1(X^\circ) \;\rightarrow\; 0$$

Example: K3 Surfaces

From Mayer-Vietoris obtain exact subsequence:

Local Model
Defect/Symmery Ops

$$0 \to H_2(X^\circ) \to H_2(X) \to H_1(\partial X^{\mathrm{loc}}) \to H_1(X^\circ) \to 0$$

Example: K3 Surfaces

From Mayer-Vietoris obtain exact subsequence:

$$0 \to H_2(X^\circ) \to \underbrace{H_2(X)}_{\substack{\text{Massive} \\ \text{Matter}}} \to \underbrace{H_1(\partial X^{\mathrm{loc}})}_{\substack{\text{Local Model} \\ \text{Defect/Symmery Ops}}} \to H_1(X^\circ) \to 0$$

Example: K3 Surfaces

From Mayer-Vietoris obtain exact subsequence:

$$0 \to \underset{\substack{\vdots \\ \text{Extra } U(1)\text{'s}}}{H_2(X^\circ)} \to \underset{\substack{\vdots \\ \text{Massive Matter}}}{H_2(X)} \to \underset{\substack{\vdots \\ \text{Local Model Defect/Symmery Ops}}}{H_1(\partial X^{\mathrm{loc}})} \to H_1(X^\circ) \to 0$$

Example: K3 Surfaces

From Mayer-Vietoris obtain exact subsequence:

$$\begin{array}{ccccccccccc} & & \text{Extra } U(1)\text{'s} & & \begin{array}{c}\text{Massive}\\ \text{Matter}\end{array} & & \begin{array}{c}\text{Local Model}\\ \text{Defect/Symmery Ops}\end{array} & & \begin{array}{c}\text{Emergent/Broken}\\ \text{Symmetries}\end{array} & & \\ & & \vdots & & \vdots & & \vdots & & \vdots & & \\ 0 & \rightarrow & H_2(X^\circ) & \rightarrow & H_2(X) & \rightarrow & H_1(\partial X^{\mathrm{loc}}) & \rightarrow & H_1(X^\circ) & \rightarrow & 0 \end{array}$$

$$\mathrm{Tor}\, H_2(X)^\vee \cong \mathrm{Tor}\, H_1(X^\circ)$$

Example: K3 Surfaces

From Mayer-Vietoris obtain exact subsequence:

$$
\begin{array}{ccccccccccc}
 & & \text{Extra } U(1)\text{'s} & & \begin{array}{c}\text{Massive}\\ \text{Matter}\end{array} & & \begin{array}{c}\text{Local Model}\\ \text{Defect/Symmery Ops}\end{array} & & \begin{array}{c}\text{Emergent/Broken}\\ \text{Symmetries}\end{array} & & \\
 & & \vdots & & \vdots & & \vdots & & \vdots & & \\
0 & \to & H_2(X^\circ) & \to & H_2(X) & \to & H_1(\partial X^{\text{loc}}) & \to & H_1(X^\circ) & \to & 0 \\
 & & & & & \vdots & & & & & \\
 & & & & & \begin{array}{c}\text{Gauging/Trivializing}\\ \text{Symmetries}\end{array} & & & & &
\end{array}
$$

$$\text{Tor } H_2(X)^\vee \cong \text{Tor } H_1(X^\circ)$$

Example: K3 Surfaces

From Mayer-Vietoris obtain exact subsequence:

Extra $U(1)$'s

Massive Matter

Local Model Defect/Symmery Ops

Emergent/Broken Symmetries

$$0 \to H_2(X^\circ) \to H_2(X) \to H_1(\partial X^{\text{loc}}) \to H_1(X^\circ) \to 0$$

Gauging/Trivializing Symmetries

Bulk comparison of Symmetries

$$\text{Tor}\, H_2(X)^\vee \cong \text{Tor}\, H_1(X^\circ)$$

Example: K3 Surfaces

From Mayer-Vietoris obtain exact subsequence:

Extra $U(1)$'s

Massive Matter

Local Model Defect/Symmery Ops

Emergent/Broken Symmetries

$$0 \rightarrow H_2(X^\circ) \rightarrow H_2(X) \rightarrow H_1(\partial X^{\text{loc}}) \rightarrow H_1(X^\circ) \rightarrow 0$$

$U(1)$ Charge Normalization

Gauging/Trivializing Symmetries

Bulk comparison of Symmetries

$$\text{Tor}\, H_2(X)^\vee \cong \text{Tor}\, H_1(X^\circ)$$

Example: K3 Surfaces

Fate of generalized global symmetries? No global symmetries?

Explicitly: no global symmetries is equivalent to extactness of the sequence

$$0 \;\to\; H_2(X)/\mathrm{Im}\,\jmath_2 \;\xrightarrow{\partial_2}\; H_1(\partial X^{\mathrm{loc}}) \cong \oplus_i\, H_1(\partial X_i^{\mathrm{loc}}) \;\xrightarrow{\imath_1}\; H_1(X^\circ) \;\to\; 0$$

Comment: all groups are pure torsion (for K3 examples).

Example: K3 Surfaces

Let $X = T^4/\mathbb{Z}_n$ with $n = 2, 3, 4, 6$. Evaluate

$$0 \to H_2(X^\circ) \xrightarrow{\jmath_2} H_2(X) \xrightarrow{\partial_2} H_1(\partial X^{\mathrm{loc}}) \xrightarrow{\imath_1} H_1(X^\circ) \to 0$$

(See paper for non-abelian quotients of tori + computations)

$$T^4/\mathbb{Z}_2 : \quad 0 \xrightarrow{\imath_2} \mathbb{Z}^6 \xrightarrow{\jmath_2} \mathbb{Z}^6 \oplus \mathbb{Z}_2^5 \xrightarrow{\partial_2} \mathbb{Z}_2^{16} \xrightarrow{\imath_1} \mathbb{Z}_2^5 \xrightarrow{\jmath_1} 0$$

$$T^4/\mathbb{Z}_3 : \quad 0 \xrightarrow{\imath_2} \mathbb{Z}^4 \xrightarrow{\jmath_2} \mathbb{Z}^4 \oplus \mathbb{Z}_3^3 \xrightarrow{\partial_2} \mathbb{Z}_3^{9} \xrightarrow{\imath_1} \mathbb{Z}_3^3 \xrightarrow{\jmath_1} 0$$

$$T^4/\mathbb{Z}_4 : \quad 0 \xrightarrow{\imath_2} \mathbb{Z}^4 \xrightarrow{\jmath_2} \mathbb{Z}^4 \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_2^2 \xrightarrow{\partial_2} \mathbb{Z}_4^4 \oplus \mathbb{Z}_2^6 \xrightarrow{\imath_1} \mathbb{Z}_4 \oplus \mathbb{Z}_2^2 \xrightarrow{\jmath_1} 0$$

$$T^4/\mathbb{Z}_6 : \quad 0 \xrightarrow{\imath_2} \mathbb{Z}^4 \xrightarrow{\jmath_2} \mathbb{Z}^4 \oplus \mathbb{Z}_6 \xrightarrow{\partial_2} \mathbb{Z}_6 \oplus \mathbb{Z}_3^4 \oplus \mathbb{Z}_2^5 \xrightarrow{\imath_1} \mathbb{Z}_6 \xrightarrow{\jmath_1} 0$$

Computations building on: [Spanier, 1956], [Nikulin, 1975], [Shioda, Inose, 1977], [Nahm, Wendland, 2001], [Wendland, 2002]

Example: K3 Surfaces

In particular the sequence

$$0 \rightarrow H_2(X^\circ) \xrightarrow{\jmath_2} H_2(X) \xrightarrow{\partial_2} H_1(\partial X^{\text{loc}}) \xrightarrow{\imath_1} H_1(X^\circ) \rightarrow 0$$

determines the gauge group G:

$$T^4/\mathbb{Z}_2 : \qquad G = \frac{(SU(2)^{16}/\mathbb{Z}_2^5) \times U(1)^6}{\mathbb{Z}_2^6}$$

$$T^4/\mathbb{Z}_3 : \qquad G = \frac{(SU(3)^{9}/\mathbb{Z}_3^3) \times U(1)^4}{\mathbb{Z}_3^3}$$

$$T^4/\mathbb{Z}_4 : \qquad G = \frac{(SU(4)^{4}/\mathbb{Z}_4 \times \mathbb{Z}_2^2) \times SU(2)^6 \times U(1)^4}{\mathbb{Z}_4^2 \times \mathbb{Z}_2^2}$$

$$T^4/\mathbb{Z}_6 : \qquad G = \frac{([SU(6) \times SU(3)^4 \times SU(2)^5]/\mathbb{Z}_3 \times \mathbb{Z}_2) \times U(1)^4}{\mathbb{Z}_6^3 \times \mathbb{Z}_2}$$

Calabi-Yau Threefold Example: $T^6/\mathbb{Z}_3$

Local Models: $27 \times \mathbb{C}^3/\mathbb{Z}_3$

Local Physics: $27 \times E_0$ Seiberg 5D SCFT [Seiberg, 1996]

Cutting and gluing gives two exact sequences:

$$0 \to H_4(X^\circ) \xrightarrow{\jmath_4} H_4(X) \xrightarrow{\partial_4} H_3(\partial X^{\text{loc}}) \xrightarrow{\imath_3} Tor\, H_3(X^\circ) \to 0$$

$$0 \to \mathbb{Z}^9 \xrightarrow{\jmath_4} \mathbb{Z}^9 \oplus \mathbb{Z}_3^4 \xrightarrow{\partial_4} \mathbb{Z}_3^{27} \xrightarrow{\imath_3} \mathbb{Z}_3^{17} \to 0$$

$$0 \to H_2(X^\circ) \xrightarrow{\jmath_2} H_2(X) \xrightarrow{\partial_2} H_1(\partial X^{\text{loc}}) \xrightarrow{\imath_1} \text{Tor}\, H_1(X^\circ) \to 0$$

$$0 \to \mathbb{Z}^9 \xrightarrow{\jmath_2} \mathbb{Z}^9 \oplus \mathbb{Z}_3^{17} \xrightarrow{\partial_2} \mathbb{Z}_3^{27} \xrightarrow{\imath_1} \mathbb{Z}_3^4 \to 0\,.$$

No global symmetries?

Starting Point: electric frame $\rightarrow \mathbb{Z}_3^{27}$ 1-form symmetry group

Defect operators and Symmetry operators

$$0 \rightarrow H_4(X^\circ) \xrightarrow{\jmath_4} H_4(X) \xrightarrow{\partial_4} H_3(\partial X^{\text{loc}}) \xrightarrow{\imath_3} Tor\, H_3(X^\circ) \rightarrow 0$$

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Starting Point: electric frame $\to \mathbb{Z}_3^{27}$ 1-form symmetry group

Defect operators and Symmetry operators

Add matter and break symmetries

$$0 \to H_4(X^\circ) \xrightarrow{\jmath_4} H_4(X) \xrightarrow{\partial_4} H_3(\partial X^{\text{loc}}) \xrightarrow{\imath_3} Tor\, H_3(X^\circ) \to 0$$

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Consider purely electric frame: $\mathbb{Z}_3^{27}$ 1-form symmetry group

Defect operators and Symmetry operators:

Add matter and break symmetries

Gauge/trivialize symmetries and obtain dual quantum symmetry

$$0 \to H_4(X^\circ) \xrightarrow{\jmath_4} H_4(X) \xrightarrow{\partial_4} H_3(\partial X^{\mathrm{loc}}) \xrightarrow{\imath_3} Tor\, H_3(X^\circ) \to 0$$

$$0 \to \mathbb{Z}^9 \xrightarrow{\jmath_4} \mathbb{Z}^9 \oplus \mathbb{Z}_3^4 \xrightarrow{\partial_4} \mathbb{Z}_3^{27} \xrightarrow{\imath_3} \mathbb{Z}_3^{17} \to 0$$

$$0 \to H_2(X^\circ) \xrightarrow{\jmath_2} H_2(X) \xrightarrow{\partial_2} H_1(\partial X^{\mathrm{loc}}) \xrightarrow{\imath_1} \mathrm{Tor}\, H_1(X^\circ) \to 0$$

$$0 \to \mathbb{Z}^9 \xrightarrow{\jmath_2} \mathbb{Z}^9 \oplus \mathbb{Z}_3^{17} \xrightarrow{\partial_2} \mathbb{Z}_3^{27} \xrightarrow{\imath_1} \mathbb{Z}_3^4 \to 0\,.$$

Starting Point: electric frame $\to \mathbb{Z}_3^{27}$ 1-form symmetry group

Defect operators and Symmetry operators

Add matter and break symmetries

Gauge/trivialize symmetries and obtain dual quantum symmetry

Add branes and break dual quantum symmetry

$$0 \to H_4(X^\circ) \xrightarrow{\jmath_4} H_4(X) \xrightarrow{\partial_4} H_3(\partial X^{\text{loc}}) \xrightarrow{\imath_3} \text{Tor}\, H_3(X^\circ) \to 0$$

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$$0 \to \mathbb{Z}^9 \xrightarrow{\jmath_2} \mathbb{Z}^9 \oplus \mathbb{Z}_3^{17} \xrightarrow{\partial_2} \mathbb{Z}_3^{27} \xrightarrow{\imath_1} \mathbb{Z}_3^4 \to 0\,.$$

Resulting gauge group: $G = \mathbb{Z}_3^{17} \times U(1)^9$

Omissions and Outlook

We further discuss

- Elliptic CY_2 and CY_3 Examples
- Graphs of Symmetry Topological Field Theories
- Trivializations of 2-groups ($\rightarrow$ Jonathan's Talk, $T^6/\mathbb{Z}_4$)

Thank you for your time.