Higher Symmetries via Cutting and Gluing of Orbifolds

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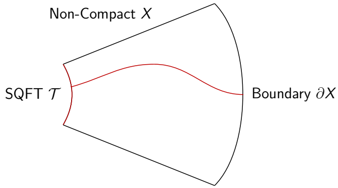
Simons Collaboration, Sixth Annual Meeting

September 9th, 2022

Motivation

- Consider IIA/IIB/**M**/F-theory on geometry $\mathbb{R}^{1,d-1} \times X$
- Let X be non-compact, singular and of special holonomy
- $\Rightarrow$ supersymmetric quantum field theory $\mathcal{T}_X$
- What to study about such theories? Symmetries!

  [Del Zotto, Heckman, Park, Rudelius, 2015], [Albertini, Del Zotto, García Etxebarria, Hosseini, 2020], [Morrison, Schäfer-Nameki, Willet, 2020], ...



- CY cones, elliptically CY-threefolds, G_2-manifolds [Cvetič, Heckman, MH, Torres, 2022], [MH, Morrison, Schäfer-Nameki, 2022], [Del Zotto, García Etxebarria, Schäfer-Nameki, 2022], ...

Motivation

- Tools: Geometric engineering dictionary
  $\Rightarrow$ Physics of $\mathcal{T}_X$ is filtered by mathematical structure of X
- Metric data, very important, but hard [Joyce, 1996], [Atiyah, Witten, 2001], [Acharya, Witten, 2001], [Acharya, 2000], [Kovalev, 2003], [Corti, Haskins, Nordström, Pacini, 2015], ...
- Topological data, still very important, but easy [Cvetič, Heckman, MH, Torres, 2022], [Del Zotto, García Etxebarria, Schäfer-Nameki, 2022], [MH, Morrison, Schäfer-Nameki, Wang, 2022]

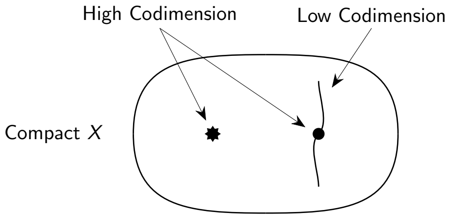
Punchline: [Gaiotto, Kapustin, Seiberg, Willett, 2014]

Symmetries are generated by topological operators!

- Objective: study symmetries of $\mathcal{T}_X$ via the topology of X

Motivation

- Let X be **compact**, singular and of special holonomy
- $\Rightarrow$ supergravity theory $\mathcal{S}_X$
- What can we say about such theories? [Apruzzi, Dierigl, Lin, 2020], [Cvetič, Dierigl, Lin, Zhang, 2020], [Heidenreich, McNamara, Montero, Reece, Rudelius 2021], . . .



Motivation

- What about $\mathcal{S}$?
- Localized Sectors: $\mathcal{S} \supset \mathcal{T}_1 \otimes \mathcal{T}_2 \otimes \mathcal{T}_3 \otimes \dots$

- Local limits: $\mathcal{S} \twoheadrightarrow \mathcal{T}_k$ [Beasley, Heckman, Vafa, 2008], [Pantev, Wijnholt, 2009], ...
- Symmetries emerge in local limits

- Alternatively: embeddings $\mathcal{T}_k \hookrightarrow \mathcal{S}$
- Symmetries are broken or gauged

  [Banks, Seiberg, 2011], [Apruzzi, Dierigl, Lin, 2020], [Braun, Larfors, Öhlman, 2021], ...

Punchline: **Topology determines emergence/gauging/breaking**

$\Rightarrow$ Come to my talk at the workshop next week!

Defects of $\mathcal{T}_X$

- Consider M-theory on non-compact X
- Wrap M2 or M5 branes on non-compact cycles [Albertini, Del Zotto, García Etxebarria, Hosseini, 2020], [Morrison, Schäfer-Nameki, Willet, 2020]

$$\frac{H_{k+1}(X,\partial X)}{H_{k+1}(X)} \cong H_k(\partial X)|_{\text{triv}}$$

constructing defects $\mathfrak{D}^{\text{M2}}_{2-k}(\gamma_k)$ or $\mathfrak{D}^{\text{M5}}_{5-k}(\sigma_k)$

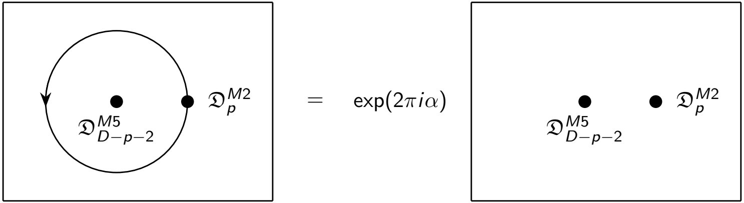
- Defects are non-dynamical $(2-k)$ or $(5-k)$ dimensional electric or magnetic objects in $\mathcal{T}$ of infinite mass/tension
- Collect all defects into the defect group $\mathfrak{D}$ [Del Zotto, Heckman, Park, Rudelius, 2015],

$$\mathfrak{D} = \bigoplus_m \mathfrak{D}^{(m)} \quad \text{with} \quad \mathfrak{D}^{(m)} = \bigoplus_{p-k=m} \frac{H_{k+1}(X,\partial X)}{H_{k+1}(X)}$$

- Group operation is fusion of defects

Properties of Defects

- The theory $\mathcal{T}_X$ implicitly assumes a selection of defects
- Phase ambiguity in correlation functions [Seiberg, Taylor, 2011]



with $\alpha = \langle \mathfrak{D}_p^{M2}, \mathfrak{D}_{D-p-2}^{M5} \rangle \in \mathbb{Q}/\mathbb{Z}$

- Phase α given by the linking of cycles wrapped by the M2, M5
- Polarizations $\Lambda^\vee \subset \mathfrak{D}$ determine absolute theories [Gaiotto, Moore, Neitzke, 2010], [Aharony, Seiberg, Tachikawa, 2013], [Gukov, Hsin, Pei, 2020]

Symmetry Operators

- Wrap M2 or M5 branes on cycles at infinity, [Heckman, MH, Torres, Zhang, yesterday]

$$\gamma_\ell, \sigma_\ell \in H_\ell(\partial X)$$

constructing symmetry operators $\mathcal{U}^{\text{M2}}_{3-\ell}(\gamma_\ell)$ or $\mathcal{U}^{\text{M5}}_{6-\ell}(\sigma_\ell)$.

- Symmetry operators are complicated [Freed, Moore, Segal, 2006], [García Etxebarria, Heidenreich, Regalado, 2019]

$$\mathcal{U}^{\text{M2}}_{3-\ell}(\gamma_\ell) = \exp(\mathcal{S}^{M2}_{\text{top}}) = \exp\left(2\pi i \int_{\Sigma_{3-\ell}\times\gamma_\ell} \breve{G}_4 + \dots\right)$$
$$\mathcal{U}^{\text{M5}}_{6-\ell}(\sigma_\ell) = \exp(\mathcal{S}^{M5}_{\text{top}}) = \exp\left(2\pi i \int_{\Sigma_{6-\ell}\times\gamma_\ell} \breve{G}_7 + \dots\right)$$

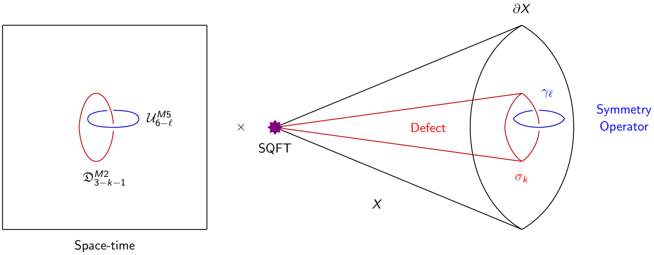
Excursion

- For example, for D3 branes we have $\mathcal{U} = \exp(\mathcal{S}_{\text{top}}^{D3})$ with [Minasian, Moore, 1997]

$$\mathcal{S}_{\text{top}}^{D3} = 2\pi i \int_{\Sigma} \exp(\mathcal{F}_2) \sqrt{\frac{\widehat{A}(T\Sigma)}{\widehat{A}(N\Sigma)}} \, (C_0 + C_2 + C_4)$$

Action of Symmetry Operators

- Phase ambiguity $\leftrightarrow$ flux operators do not commute [Seiberg, Taylor, 2011], [García Etxebarria, Heidenreich, Regalado, 2019]
- Flux operator action on defects is determined by linking $\mathrm{Link}(\gamma_\ell, \sigma_k)$ [García Etxebarria, 2022], [Heckman, MH, Torres, Zhang, 2022]



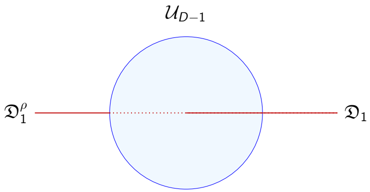
- Pontryagin dual group Λ of n-form symmetries
- For figure $n = 3 - k - 1$

Action of Symmetry Operators

- n-form symmetries: $\mathcal{U}_p : \mathfrak{D}_n \to \mathfrak{D}_n$

  [Gaiotto, Kapustin, Seiberg, Willett, 2014], [Sharpe, 2015]
- In space-time dimension $D = p + n + 1$
- $\mathcal{U}_p$ can act on defects $\mathfrak{D}_{n'}$ with $n' \geq n$

  [Hsin, Lam, Seiberg 2018], [Benini, Cordova, Hsin 2018], ...
- Example: $\mathcal{U}_{D-1}$ can act on lines



Generalizations, Comments and Summary

- Brane perspective gives **fusion rules** of symmetry operators
- Generalizations to **non-invertible symmetries** naturally covered
- Prediction: further **generalizations** of 'Symmetry'
- Consequence: symmetries **trivialize** in compact models

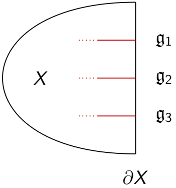
Higher symmetry data:

- $\{$ Topological defect operators $\mathcal{U}$ $\}$
- $+$ Representations (Defects $\mathfrak{D}$)
- $+$ Fusion algebra

2-Group Symmetries via Geometry

2-Group Symmetries via Geometry

- n-form symmetries can mix
  [Benini, Cordova, Hsin 2018], [Cordova, Dumitrescu, Intriligator 2018], ...
- 2-group symmetry: mixing of 0-form and 1-form symmetries
  [Apruzzi, Bhardwaj, Gould, Schäfer-Nameki, 2021], [Apruzzi, Bhardwaj, Schäfer-Nameki, Oh, 2021], [Cvetič, Heckman, MH, Torres, 2022], [Del Zotto, García Etxebarria, Schäfer-Nameki, 2022]
- Concrete geometric context we consider:



$$\widetilde{G} = \widetilde{G}_1 \times \widetilde{G}_2 \times \widetilde{G}_3 \times \dots$$

$\widetilde{G}_i$ simply connected with algebra $\mathfrak{g}_i$

- ADE singularities in the boundary ∂X
- Naive 0-form center symmetry $Z_{\widetilde{G}}$

Line Defects $\widetilde{A}^\vee$

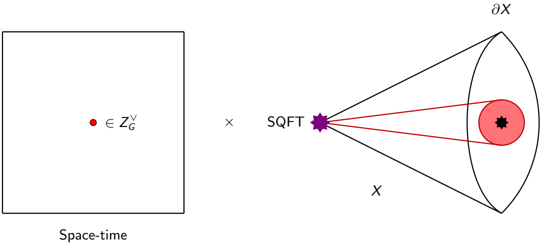
- ∂X is singular!
- Singular locus K, tube ∂X^{loc}, $\partial X^\circ = \partial X \setminus K$
- Generalize to orbifold homology: $H_* \to H_*^{\text{orb}}$ [Moerdijk, Pronk, 2003]
- $\Rightarrow$ new homology classes

- Wrap M2 branes on $\gamma_1 \in H_1(\partial X)$, generate defects $\mathcal{A}^\vee$
- Wrap M2 branes on $\widetilde{\gamma}_1 \in H_1^{\text{orb}}(\partial X)$, generate defects $\widetilde{\mathcal{A}}^\vee$
- $\Rightarrow$ projection of lines $\widetilde{\mathcal{A}}^\vee \to \mathcal{A}^\vee$

- Physics: $\widetilde{\mathcal{A}}^\vee$ are line operators modulo local operator interfaces with faithful action under flavor symmetry group G [Lee, Ohmori, Tachikawa, 2021]

Local Defects $Z_G^\vee$

- Observation: $H_1^{\text{orb}}(\partial X) = H_1(\partial X^\circ)$ [Moerdijk, Pronk, 2003]
- Interpretation:



- Wrap M2 brane on Disk $\times \mathbb{R}_+ \to$ Local operators in $Z_{\widetilde{G}}^\vee$

2-Groups

- 2-group of defects:

$$0 \to Z_G^\vee \to Z_{\widetilde{G}}^\vee \to \widetilde{\mathcal{A}}^\vee \to \mathcal{A}^\vee \to 0$$

- Geometrifies to (Mayer-Vietoris sequence, cover $\partial X^{\text{loc}} \cup \partial X^\circ$)

  [Cvetič, Heckman, MH, Torres, 2022]

$$0 \to \ker \iota_1 \to H_1(\partial(\partial X^\circ)) \xrightarrow{\iota_1} H_1(\partial X^\circ) \to H_1(\partial X) \to 0$$

- 2-group of symmetry operators (Pontryagin dual):

  [Kapustin, Thorngren, 2013], [Lee, Ohmori, Tachikawa, 2021]

$$0 \to \mathcal{A} \to \widetilde{\mathcal{A}} \to Z_{\widetilde{G}} \to Z_G \to 0$$

Properties of 2-Groups $(\mathcal{A}, G, P, \rho)$

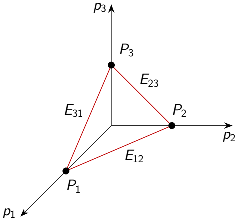
- Contain 1-form symmetry group $\mathcal{A}$
- Contain 0-form symmetry (center) group Z_G
- Isomorphism class data: Postinkov class $P \in H^3(BG, \mathcal{A})$
- Here, no action $\rho : G \to \text{Aut}(\mathcal{A})$

- Can be gauged/have anomalies
- Constrain RG flow, IR physics, vacuum structure
- Selection rules

Example: $\mathbb{C}^3/\Gamma$, $\Gamma = \mathbb{Z}_n$

- Geometry: $(z_1, z_2, z_3) \sim (\omega^{k_1} z_1, \omega^{k_2} z_2, \omega^{k_3} z_3)$
  with primitive $\omega^n = 1$ and $k_1 + k_2 + k_3 = 0$ mod n

  [Joyce, 2000], [Tian, Wang, 2021], [Del Zotto, Heckman, Meynet, Moscrop, Zhang, 2022]
- Boundary: $S^5/\Gamma = \{|z_1|^2 + |z_2|^2 + |z_3|^2 = 1\}/\Gamma$
- A-type ADE singularities: circle $|z_i| = 1$ rank $\gcd(n, k_i)$
- Toric model for ∂X with T^3 fiber and triangle base



Example: $\mathbb{C}^3/\Gamma$, $\Gamma = \mathbb{Z}_{2k}$

- Concrete $(k_1, k_2, k_3) = (1, 1, 2k-2)$
- A_1 singularity along $z_1 = z_2 = 0$
- A_1 singularity in boundary $|z_3| = 1$

- Deformation retraction $\partial X^\circ \to S^3/\mathbb{Z}_{2k}$, $H_1(\partial X^\circ) \cong \mathbb{Z}_{2k}$
- Armstrong $H_1(\partial X) \cong \mathbb{Z}_2$

$$\begin{array}{ccccccccccc} 0 & \to & \mathcal{A} & \to & \widetilde{\mathcal{A}} & \to & Z_{\widetilde{G}} & \to & Z_G & \to & 0 \\ 0 & \to & \mathbb{Z}_2 & \to & \mathbb{Z}_{2k} & \to & \mathbb{Z}_k & \to & 0 & \to & 0 \end{array}$$

- $\Rightarrow G = SU(2)/\mathbb{Z}_2 = SO(3)$

Generalizations and Comments

- We study all 5d SCFTs engineered by M-theory on $\mathbb{C}^3/\Gamma$ with $\Gamma \subset SU(3)$ and $\Gamma \cong \mathbb{Z}_n \times \mathbb{Z}_m$ with $m \,|\, n$
- For example, 5d T_N theory [Benini, Benvenuti, Tachikawa, 2009] $\Gamma \cong \mathbb{Z}_N \times \mathbb{Z}_N$

$$G = \frac{SU(N) \times SU(N) \times SU(N)}{\mathbb{Z}_N \times \mathbb{Z}_N} \quad \text{and} \quad \mathcal{A} = 0$$

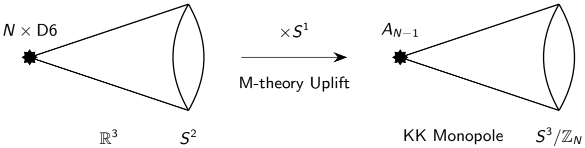
- Match charge lattice analysis [Apruzzi, Bhardwaj, Schäfer-Nameki, Oh, 2021] and Lagrangian analysis when gauge theory phases exist
- Consistent with compactifications (eg. 4d T_N theory) [Gaiotto, Maldacena, 2012], [Bhardwaj, 2021]
- Intrinsically non-Lagrangian characterization
- Minimalistic characterization

Supersymmetric D6 branes: SQCD-like theories

- Hard: metric uplift of susy D6-brane configurations

  [Foscolo, Haskins, Nordstöm, 2017], [Acharya, Foscolo, Najjar, Svanes, 2020]
- Easy: topological uplift of susy D6-brane configurations

- D6 brane in IIA is co-dimension 3 with uplift as



Uplifting Procedure

IIA Configuration:

- CY_3 X_6 with N_i D6 branes wrapped on sLag submanifold M_i
- D6-branes source RR flux $F_2 = dC_1$ counting branes $\int_{S^2} F_2 = n_{D6}$
- Expand F_2 in fluxes through cycles linking D6 loci

M-theory lift:

- Local geometry normal to N_i D6 branes lifts to $\mathbb{C}^2/\mathbb{Z}_{N_i}$
- $X_6^\circ = X_6 \setminus \cup_i M_i$ lifts to a circle bundle $X_7^\circ \to X_6^\circ$ with Euler class $e = F$
- IIA set-up lifts to a circle bundle $X_7 \to X_6$ with A_{N_i-1} loci

Uplifting Procedure

Restrict constructions to the smooth boundary ∂X_7°, Gysin sequence:

$$\cdots \to H^k(X_7^\circ) \to H^{k-1}(X_6^\circ) \xrightarrow{e\wedge} H^{k+1}(X_6^\circ) \to H^{k+1}(X_7^\circ) \to \dots$$

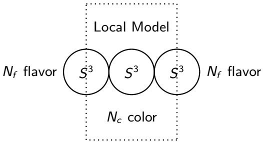
Glue local neighborhoods X_7^{loc} back in, Mayer-Vietoris sequence:

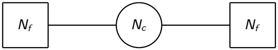
$$\cdots \to H_k(\partial X_7^\circ) \to H_k(X_7^{\text{loc}}) \oplus H_k(X_7^\circ) \to H_k(X_7) \to \dots$$

Possible extensions: Orientifold planes

Example

- CY_3 [Feng, He, Kennaway, Vafa, 2008], [Del Zotto, Oh, Zhou, 2021] with supersymmetric three-spheres
- Consider local geometry $X_6 = T^*S^3$ of a fixed (color) three-sphere
- Color S^3 intersects flavor S^3's at points
- Flavor S^3's decompactify to fiber classes topologically $\mathbb{R}^3 \subset T^*S^3$





- Flavor symmetry $G_F = [SU(N_f) \times SU(N_f)]/Z$ with center $\mathbb{Z}_{\gcd(N_f, N_c)}$

Summary and Conclusion

- Added higher symmetry structures to the geometric engineering dictionary
  - Geometrized classes of defects, $\mathcal{A}^\vee, \widetilde{\mathcal{A}}^\vee, Z^\vee_{\widetilde{G}}, Z^\vee_G$
  - Geometrized extension properties (2-group = Mayer-Vietoris)
  - Geometrized symmetry operators as branes at infinity

  all via the boundary geometries $\partial X, \partial X^\circ, \partial X^{\text{loc}}$.
- Non-Lagrangian methods, applicable to strongly coupled SQFTs
- Many avenues for furture research
  - Boundaries of boundaries of boundaries $\ldots \rightarrow n$-groups
  - Symmetries and compact geometries