Relative Defects in Relative Theories

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Geometrization of (S)QFTs in D $\leq$ 6

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Punchline

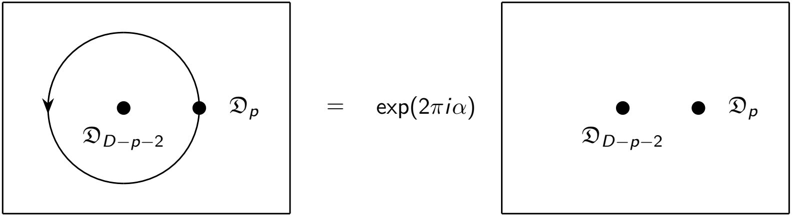
Co-dimension-two defects of 6d $\mathcal{N} = (2,0)$ theories are *relative* defects in relative theories.

[Witten, 1996], [Freed, Teleman, 2012], [Aharony, Seiberg, Tachikawa, 2013], [Gaiotto, Kapustin, Seiberg, Willet, 2014], . . . , [García Etxebarria, Heidenreich, Regalado, 2019], [Morrison, Schafer-Nameki, Willett, 2020], [Gukov, Hsin, Pei, 2020], [Braun, Larfors, Oehlmann, 2021], [Cvetic, Dierigl, Lin, Zhang, 2021], [Debray, Dierigl, Heckman, Montero, 2021], [Apruzzi, Bonetti, García Etxebarria, Hosseini, Schafer-Nameki, 2021], [Del Zotto, Heckman, Meynet, Moscrop, Zhang, 2022], . . .

Overview

Relative Theories $\mathcal{X}_D$

- Defect Group: $\mathfrak{D} = \oplus_p \mathfrak{D}_p$
- Phase ambiguity in correlation functions [Seiberg, Taylor, 2011]



with $\alpha = \langle \mathfrak{D}_p, \mathfrak{D}_{D-p-2} \rangle$.

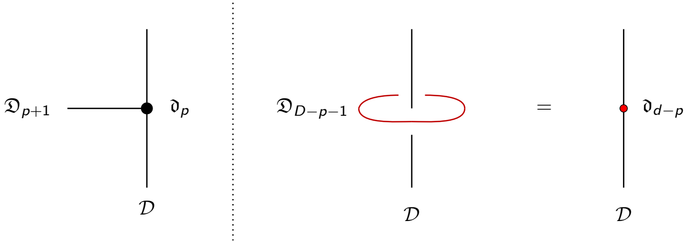
- Polarizations $\Lambda \subset \mathfrak{D}$ determine absolute theories [Gaiotto, Moore, Neitzke, 2010], [Aharony, Seiberg, Tachikawa, 2013], [Gukov, Hsin, Pei, 2020]

Relative Defects $\mathcal{D}_d \subset \mathcal{X}_D$

- Relative Defects $\mathcal{D}$ support sub-defects: $\mathfrak{d} = \oplus_p \mathfrak{d}_p$
- Natural maps between (sub)defect groups

$$\pi_p : \mathfrak{d}_p \to \mathfrak{D}_{p+1} \,, \qquad s_{D-p-1} : \mathfrak{D}_{D-p-1} \to \mathfrak{d}_{d-p}$$

referred to as projection and contraction.



- Polarizations $\lambda \subset \mathfrak{d}$ determine absolute defects
- Maps π, s fix consistency conditions betwen polarizations

$$\pi(\lambda) \subset \Lambda\,, \qquad s(\Lambda) \subset \lambda\,.$$

- The pair (λ, Λ) determines the extended operators and fixes the absolute theory
- The pair (λ, Λ) determine the bulk and defect higher form symmetries

$$\widehat{\Lambda}_p = \mathfrak{D}_p / \Lambda_p\,, \qquad \widehat{\lambda}_q = \mathfrak{d}_q / \lambda_q$$

6d $\mathcal{N} = (2,0)$ Theory

- Bulk defect group $\mathfrak{D} = \mathfrak{D}_2$ of surface defects [Del Zotto, Heckman, Park, Rudelius, 2015]
- Co-dimension-two relative defect $\mathcal{D}$
- Possible sub-defects $\mathfrak{d} = \mathfrak{d}_0 \oplus \mathfrak{d}_1 \oplus \mathfrak{d}_2$
- Non-trivial maps

$$\pi_1 : \mathfrak{d}_1 \to \mathfrak{D}_2 \,, \qquad s_2 : \mathfrak{D}_2 \to \mathfrak{d}_1$$

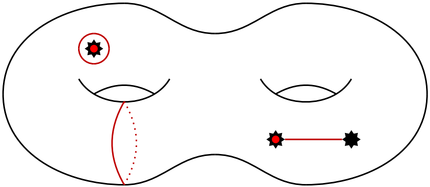
- Trapped lines

$$\mathfrak{d}_1^T = \frac{\ker \pi_1}{\operatorname{im} s_2}$$

Class S

- Compactify on Riemann surface C
- Co-dimension-two defect $\mathcal{D}$ are now punctures $\mathcal{P}$ [Xie, Wang, 2015]
- Surfaces $\mathfrak{D}_2$ wrapped on one-cycles give lines in 4d [Drukker, Morrison, Okuda, 2009], [Tachikawa, 2014], [Bhardwaj, H, Schäfer-Nameki, 2021]
- Trapped lines $\mathfrak{d}_1^T$ descend to lines in 4d

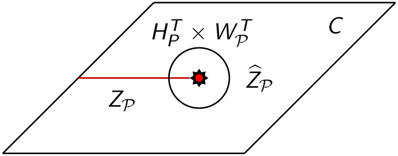
C :



Defect Group of a Puncture

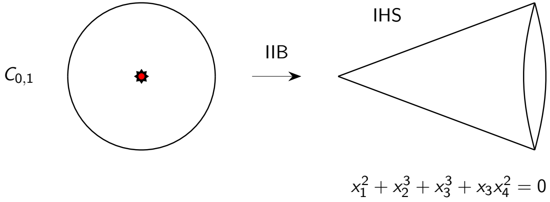
- Puncture characterized by 4d lines $\mathcal{L}_\mathcal{P} = \mathcal{W}_\mathcal{P} \times \mathcal{H}_\mathcal{P}$
- Lines $\mathcal{L}_\mathcal{P}$ determined by $\mathcal{H}_\mathcal{P}$ as $\mathcal{W}_\mathcal{P} = \widehat{\mathcal{H}}_\mathcal{P}$
- Lines trapped at puncture $\mathfrak{d}_1^T = \mathcal{W}_\mathcal{P}^T \times \mathcal{H}_\mathcal{P}^T$
- Lines from bulk surface defects $\mathcal{Z}_\mathcal{P}, \widehat{\mathcal{Z}}_\mathcal{P}$
- Short exact sequences for puncture

$$0 \to \mathcal{H}_\mathcal{P}^T \to \mathcal{H}_\mathcal{P} \to \mathcal{Z}_\mathcal{P} \to 0$$
$$0 \to \widehat{\mathcal{Z}}_\mathcal{P} \to \mathcal{W}_\mathcal{P} \to \mathcal{W}_\mathcal{P}^T \to 0$$



Evidence for $\mathcal{H}_{\mathcal{P}}^T, \mathcal{W}_{\mathcal{P}}^T \neq 0$ (Defects in Defects)

Argyres-Douglas theories. For example $AD[A_2, D_4]$.

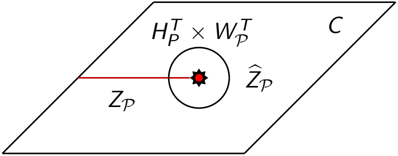


Compute defect group in IIB from Calabi-Yau cone X [Del Zotto, García-Etxebarria, Hosseini, 2020], [Closset, Schäfer-Nameki, Wang, 2020]

$$\mathfrak{d}_1 \cong \mathfrak{D}_1^{(AD)} \cong \frac{H_3(X, \partial X)}{H_3(X)} \cong H_2(\partial X) \cong \mathbb{Z}_2^2$$

Spectral Covers and Genuine Lines

- Concentrate on line defects $\mathcal{L}_{\mathcal{P}} = W_{\mathcal{P}} \times H_{\mathcal{P}}$ associated with a single puncture $\mathcal{P}$.



- Genuine lines are lines that do not arise at the end of bulk surface defects

$$\mathcal{L}_{\mathcal{P}}^{(\text{genuine})} = W_{\mathcal{P}} \times H_{\mathcal{P}}^T$$

- Poles of class S Higgs field ϕ (bulk Lie algebra $\mathfrak{g}$) characterize punctures $\mathcal{P}$ [Nardoni's Talk]
- Spectral curve (wrt rep. **R**)

$$\Sigma_{\mathbf{R}} \; : \quad \{(z, \lambda_z) \in K_C \,|\, \det(\lambda_z - \phi_{\mathbf{R}}(z)) = 0\} \subset K_C$$

- Sheets of Σ labelled by weights w_i of **R** [Longhi, Park, 2016]

$$\lambda_{z,i} = v_i(z) \frac{dz}{z} = \langle w_i, \phi_{\mathbf{R}}(z) \rangle$$

- Monodromy about $\mathcal{P}$ permutes sheets of Σ

$$\lambda_{z,i} \to \lambda_{z,j}$$

Trivial ramification at regular punctures.

- Monodromy about $\mathcal{P}$ permutes weights

$$w_i \to w_j$$

- Action on weight lattice Λ_{weights} gives action on root lattice

$$M_{\mathcal{P}} \; : \quad \Lambda_{\text{root}} \to \Lambda_{\text{root}}$$

- Genuine lines

$$\mathcal{L}_{\mathcal{P}}^{(\text{genuine})} \cong \text{Tor}\,\text{coker}\, M_{\mathcal{P}} - 1$$

Example: 4d $\mathcal{N} = 2$ SYM with $\mathfrak{g} = \mathfrak{su}(n)$

- Seiberg-Witten curve

$$v^n + u_2 v^{n-2} + \cdots + u_n - \Lambda^n(z + 1/z) = 0$$

- Consider puncture at $z = 0$, leading order

$$v^n = \frac{\Lambda^n}{z}$$

- Monodromy $z \to e^{2\pi i} z$ permutes sheets and weights v_i, w_i cyclically
- Simple roots $\alpha_i = w_i - w_{i+1}$ permuted as

$$\alpha_i \to \alpha_{i+1}\,, \quad \alpha_{n-1} \to -\sum_{i=1}^{n-1} \alpha_i$$

- $\mathcal{L}_{\mathcal{P}}^{(\text{genuine})} \cong \mathbb{Z}_n$ (Electric Wilson Line)

ALE Fibrations in IIB

- Spectral curve Σ dualizes to ALE fibration

$$\widetilde{\mathbb{C}^2/\Gamma_{\text{ADE}}} \hookrightarrow X_3 \to C_{g,n}$$

- Boundary model for puncture

$$\widetilde{\mathbb{C}^2/\Gamma_{\text{ADE}}} \hookrightarrow \partial X_3^{\mathcal{P}} \to S^1_{\mathcal{P}}$$

- Line defects from D3 branes

$$\text{Tor}\, H_2(\partial X_3^{\mathcal{P}}, \mathbb{Z}) \cong \text{Tor}\,\text{coker}\, M_{\mathcal{P}} - 1 \cong \mathcal{L}_{\mathcal{P}}^{(\text{genuine})}$$

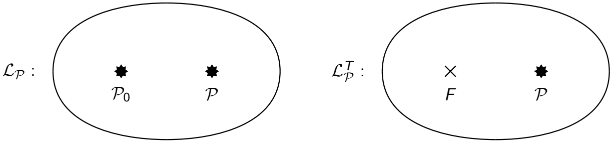
Generalized Quivers

- Motivation: The defect group of a puncture is determined by the ramification structure of the spectral cover
- Group punctures into equivalence classes according to their ramification structures
- Associate to 'nice' representatives pairs of generalized quivers

Generalized Quivers

- Derive the extension of bulk defect group by trapped defect group $\mathcal{W}_{\mathcal{P}}^T \times \mathcal{H}_{\mathcal{P}}^T$ from the quivers

$$\begin{array}{lcl} \mathcal{L}_{\mathcal{P}} & : & \mathfrak{g}_0 - M_1 - \mathfrak{g}_1 - \dots - \mathfrak{g}_{k-1} - M_k - [\mathfrak{g}_k] \\ \mathcal{L}_{\mathcal{P}}^T & : & [\mathfrak{g}_0] - M_1 - \mathfrak{g}_1 - \dots - \mathfrak{g}_{k-1} - M_k - [\mathfrak{g}_k] \end{array}$$



Omissions

- TQFT formulation [Del Zotto's Talk]
- Isolated hypersurface singularities in IIB
- New classes of irregular punctures of type A,D

Conclusion and Outlook

- We determined the line defect group for arbitrary 4d $\mathcal{N} = 2$ class S theories
- We argued for contributions to the 4d line defect group by lines trapped at irregular punctures
- Punctures (co-dimension-2 defects) are relative defects in relative theories

- Study confinement in 4d $\mathcal{N} = 1$ theories constructed by deformation of 4d $\mathcal{N} = 2$ class S theories.
- Determine 2-Groups of arbitrary class S theories